SENSITIVITY ANALYSIS OF FEEDBACK CONTROL SYSTEMS

by

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THESIS

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ABSTRACT

A technique for the use of a digital computer for simultaneous generation of feedback control system transient response and sensitivity to parameter variations is developed. The sensitivity model for a basic linear control system is a cascaded replica of the original system with the sensitivity pickoff points corresponding to the error and feedback signals of the original system. This is convenient for computer analysis with DSL/360. For systems with nonlinear elements, the sensitivity model is implemented by making the instantaneous gain equal to the slope of the nonlinear function. Sensitivity analysis is utilized for improvement of system performance, determining parameter tolerances, and predicting changes in the response due to parameter variations.

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I. INTRODUCTION

The early development of sensitivity analysis was applied to circuit theory where it is useful to know how sensitive the response of the circuit is to variations of circuit parameters such as resistance, capacitance, and inductance. There are many different definitions of sensitivity available. H. Bode [1] defines a logarithmic or normalized sensitivity function of a variable, $x_i(t,a_j)$, with respect to a parameter, a_j, as $S_{a_j}^{x_i} \equiv \frac{\partial \ell n\ a_j}{\partial \ell n\ x_i}$ and I. M. Hortowitz [2] adapted the inverse of this definition which is widely used. J. Leeds [3] presented sensitivity function for incremental changes in parameters and S. R. Parker [4] recently generalized the technique of sensitivity analysis for nonlinear circuits. Much of this work entailed the definition of R. Tomovic [5] who defined the sensitivity coefficients as $S_{a_j}^{x_i} \equiv \frac{\partial x_i}{\partial a_j}$. Kokotovic [6] adapted the definition $S_{a_j}^{x_i} \equiv \frac{\partial x_i}{\partial \ell n\ a_j}$ for use in control systems and developed a method of generating sensitivity coefficients using analog computers. In this thesis the method of Kokotovic [6] is expanded for use with digital computer techniques utilizing block diagrams and transfer functions of forward and feedback loops in control systems. The IBM/360 Digital Simulation Language (DSL) program [7] is very well suited for modeling a control system and obtaining the time response and sensitivity coefficients simultaneously.

DSL is a continuous system simulator whose non-procedural, problem-oriented input language has the attractive building-block approach of digital-analog simulators with the added power of logical and algebraic equation notation. The DSL system provides a basic set of 35 blocks from which a physical system may be modeled. Included in this set are such conventional analog components as integrators, comparators, and transfer functions. The Kokotovic [6] method of sensitivity analysis is readily adaptable to the DSL program as control systems in block-diagram form can be programed in the s-domain.

Section II of this thesis applies the Kokotovic [6] method of generating sensitivity coefficients of the basic feedback control system for the forward and feedback gain parameters. Sensitivity functions are derived and applied to a third-order system. Examples are given for the use of sensitivity coefficients in analysis, parameter adjustment for improvement of system performance, and parameter tolerance considerations. Section III applies the Kokotovic [6] method of sensitivity analysis to multiloop systems. Gain and time-constant sensitivity coefficients are derived and applied to a regulated power generator as an example. Section IV demonstrates the Kokotovic [6] method for the general case of differential equations, and Section V is devoted to the nonlinear system.

Sensitivity analysis of a control system offers an analytical method for determining those parameters of a dynamic system which have the greatest influence on the system performance. High-speed digital computers give very precise solutions for the system equations, but it is impossible to realize exact mathematical models and parameter values. Additional variations in parameters may also be caused by wear, aging, shifting of operating point, temperature, and etc. It is beneficial to know the effect of these small parameter variations on system performance. Sensitivity analysis can give this necessary insight so that all parameters of a dynamic system are not given equal importance. This is more efficient than repetitive computer solutions with parameters changed by increments. Analysis utilizing sensitivity coefficients can put emphasis on specific component tolerances and points that are critical to the performance specifications.

II. BASIC CONTROL SYSTEM SENSITIVITY

A. INTRODUCTION

The Kokotovic [6] method for computing sensitivity coefficients for various parameters is restricted to linear, time-invariant, deterministic systems. Although it is true that physical systems do not have these ideal properties, linear approximations for these systems have proven adequate in many useful cases. When parameter values are to be approximated, it is important to estimate the influence of each parameter on the system response. This can be done accurately by sensitivity analysis. Also, limits of individual parameter variations may be deduced and tolerance levels formulated.

The Kokotovic [6] method of solving for sensitivity coefficients has the advantage that all of the functions are generated simultaneously with the time-domain solution. Kokotovic [6] gives analytic proof that this is possible. As an engineering approach to analysis, it is common to approximate a high-order system by an equivalent second-order system to enable using graphs and tables for second-order response characteristics. Qualitative observations of graphs of sensitivity coefficients can give insight into the transient performance in terms of overshoot, settling time, rise time, and etc., for higher degree characteristic equations as will be seen in examples.

B. GAIN CONSTANT SENSITIVITY COEFFICIENTS

The sensitivity coefficient S_K is defined as the ratio of the change in output to the fractional change in the parameter K.

$$S_K \equiv \frac{\partial C}{\partial K/K} = \frac{\partial C}{\partial \ln K} \qquad (2.1)$$

where C is the controlled output and K is the parameter being investigated.

The canonical form of a feedback control system after Laplace transformation is shown in Fig. 2.1. The definitions of terms are as follows:

K	=	direct gain constant
KG(s)	=	direct transfer function
F	=	feedback gain constant
FH(s)	=	feedback transfer function
KG(s)FH(s)	=	loop transfer function
R(s)	=	reference input
E	=	error signal
B	=	feedback signal

The basic equations are as follows:

$$\frac{C}{R}(s) = \frac{KG(s)}{1+KG(s)FH(s)} \qquad (2.2a)$$

$$\frac{E}{R}(s) = \frac{1}{1+KG(s)FH(s)} \qquad (2.2b)$$

$$\frac{B}{R}(s) = \frac{KG(s)FH(s)}{1+KG(s)FH(s)} \qquad (2.2c)$$

In the work that follows, capital letters denote the Laplace transformation of the functions described except for the letters K and F which are constants.

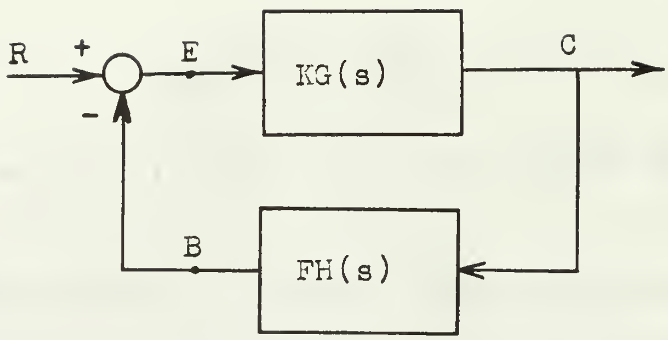


Fig.2.1. Canonical form of a feedback control system.

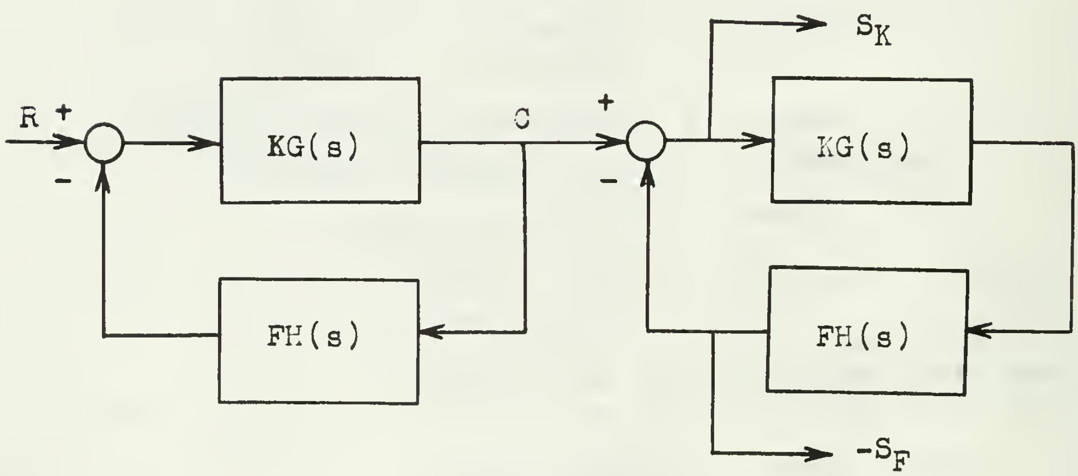


Fig.2.2. Feedback control system and sensitivity model.

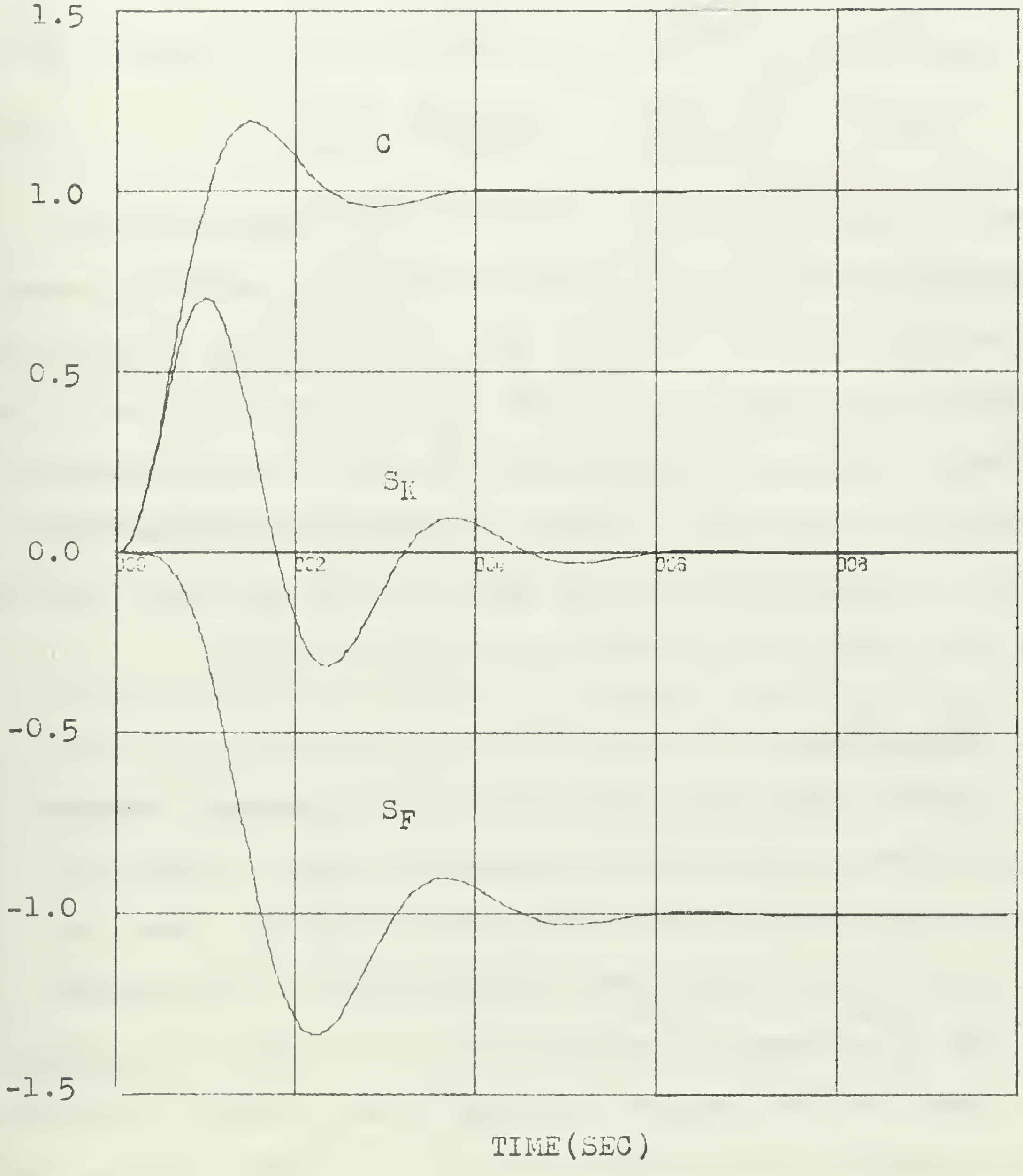


Fig.2.3. Basic control system response and sensitivity for K = 50.

Applying (2.1) to (2.2a) yields the gain sensitivity coefficients

$$S_K = \frac{RKG}{(1+KGFH)^2} = \frac{C}{1+KGFH} \tag{2.3}$$

$$S_F = \frac{-RK^2G^2FH}{(1+KGFH)^2} = \frac{C(-KGFH)}{1+KGFH} \tag{2.4}$$

These two gain sensitivity functions are shown in block diagram in Fig. 2.2. This shows that the sensitivity model is an exact replica of the original system with the controlled output as the input to the sensitivity model. The error and feedback signals of the sensitivity model are the desired sensitivity functions. It should be noted that the sensitivity model requires no new components or transfer functions and all quantities are generated simultaneously.

C. APPLICATION

In this thesis the equations for the feedback systems are programed on the IBM 360 computer using the Digital Simulation Language (DSL) program for both the transient solution and the sensitivity coefficients. This program has the advantages of programing in the s-domain in transfer function form and the accuracy of the digital computer with graphical and numerical results. A sample program is included in Appendix A and the numerical solution and sensitivity coefficients are listed in Table 2.1. Fig. 2.3 is the graph of the output and sensitivity coefficients.

The direct transfer function is

$$KG = \frac{50}{s(s+4)(s+6)} \tag{2.5}$$

For example, data from Table 2.1 when S_K is a maximum

Time	T	=	1.0
Gain	K	=	50
Sensitivity	S_K	=	0.70863
Input	R	=	unit-step
Output	C	=	0.96456

For large variations (2.1) becomes

$$SK \simeq \frac{\Delta C}{\Delta K/K} \tag{2.6}$$

Let the gain, K, change 10%, then

$$\Delta C \simeq S_K \frac{\Delta K}{K} = 0.70863(0.1) = 0.070863$$

This shows that the output will change approximately $\pm$ 0.071 if the gain varies from 50 to 55 or 45. This can be verified from Table 2.2 for the following data:

$$K = 55$$
$$T = 1.0$$
$$C = 1.0331$$
$$C' = C + \Delta C = 0.96456 + 0.070863 = 1.03541$$

and for

$$K = 45$$
$$T = 1.0$$
$$C = 0.89133$$
$$C' = C - \Delta C = 0.96456 - 0.07086 = 0.89370$$

This shows that sensitivity coefficients for changes of gain as large as 10% produce accuracy within 0.3% when compared to incremental intervals as denoted in partial derivatives.

Another example for the use of sensitivity coefficients is to compute the gain required to reduce the peak overshoot of the system transient response to a more desirable specification of one-half its initial value. Table 2.1 gives the following data:

$$K = 50$$
$$\text{Peak overshoot} = 0.2$$
$$T = 1.5$$
$$S_K = 0.36568$$

Rearranging (2.6)

$$\Delta K \approx \frac{\Delta C K}{S_K} \tag{2.7}$$

$$K' = K - \Delta K = K(1-\frac{\Delta C}{S_K}) \tag{2.8}$$
$$= 50\ (1 - \frac{0.1}{0.36568}) \approx 36$$

Table 2.2 for K = 36 shows that the peak overshoot is reduced to 0.1 and Fig. 2.4 shows the reduction of peak overshoot by a factor of one-half.

Any other design specification, such as settling time or rise time, can be adjusted by utilization of the sensitivity coefficient. In the example it should be noted that reducing the overshoot by reduction in gain, increases the rise time by 0.4 second.

Tolerance considerations for parameters are encompassed within the definition of sensitivity coefficient.

Rearranging (2.6)

$$\text{Tolerance} = \Delta K/K = \frac{\Delta C}{S_K} \tag{2.9}$$

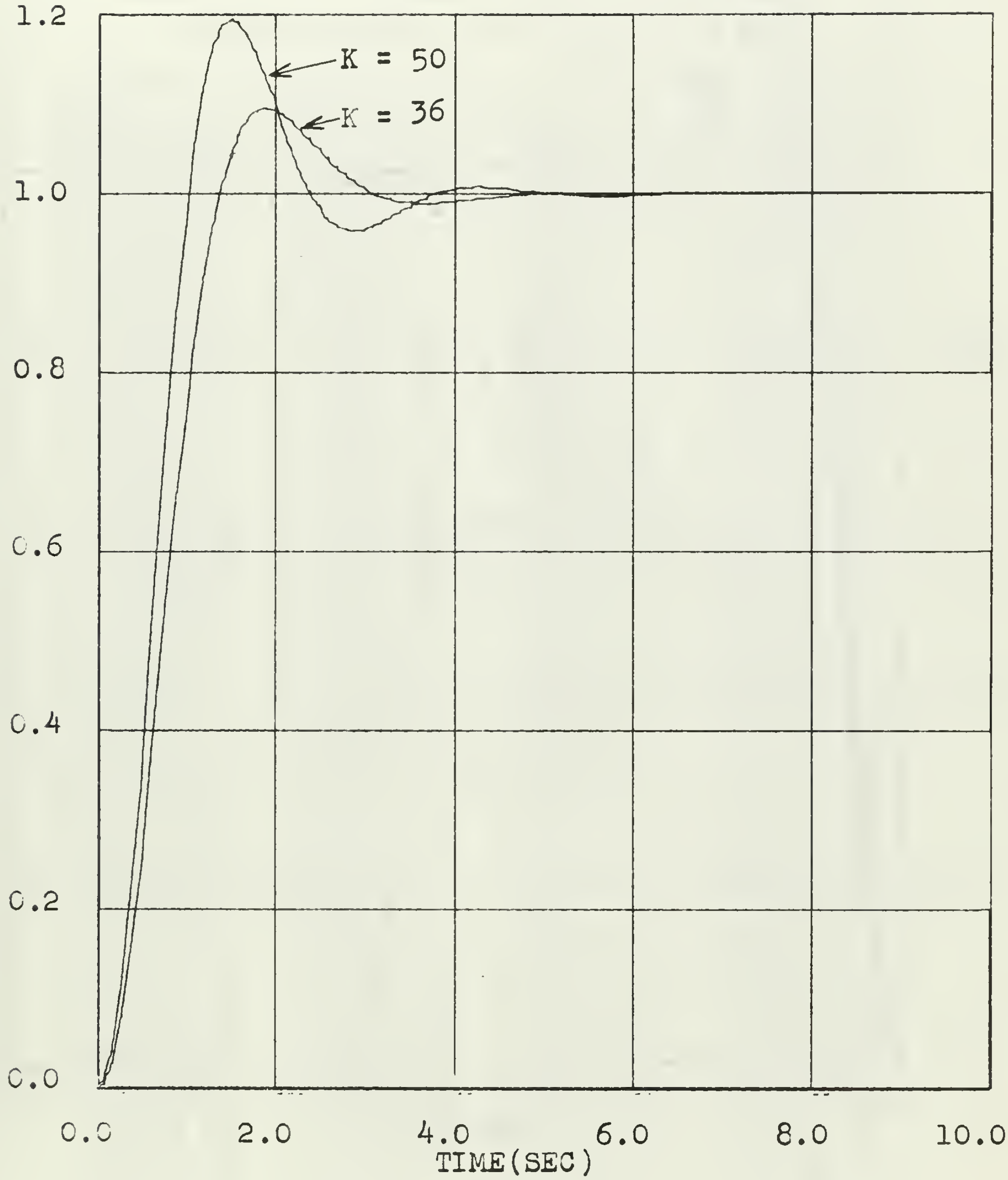


Fig.2.4. Overshoot reduction of 0.1 by utilization of sensitivity coefficients.

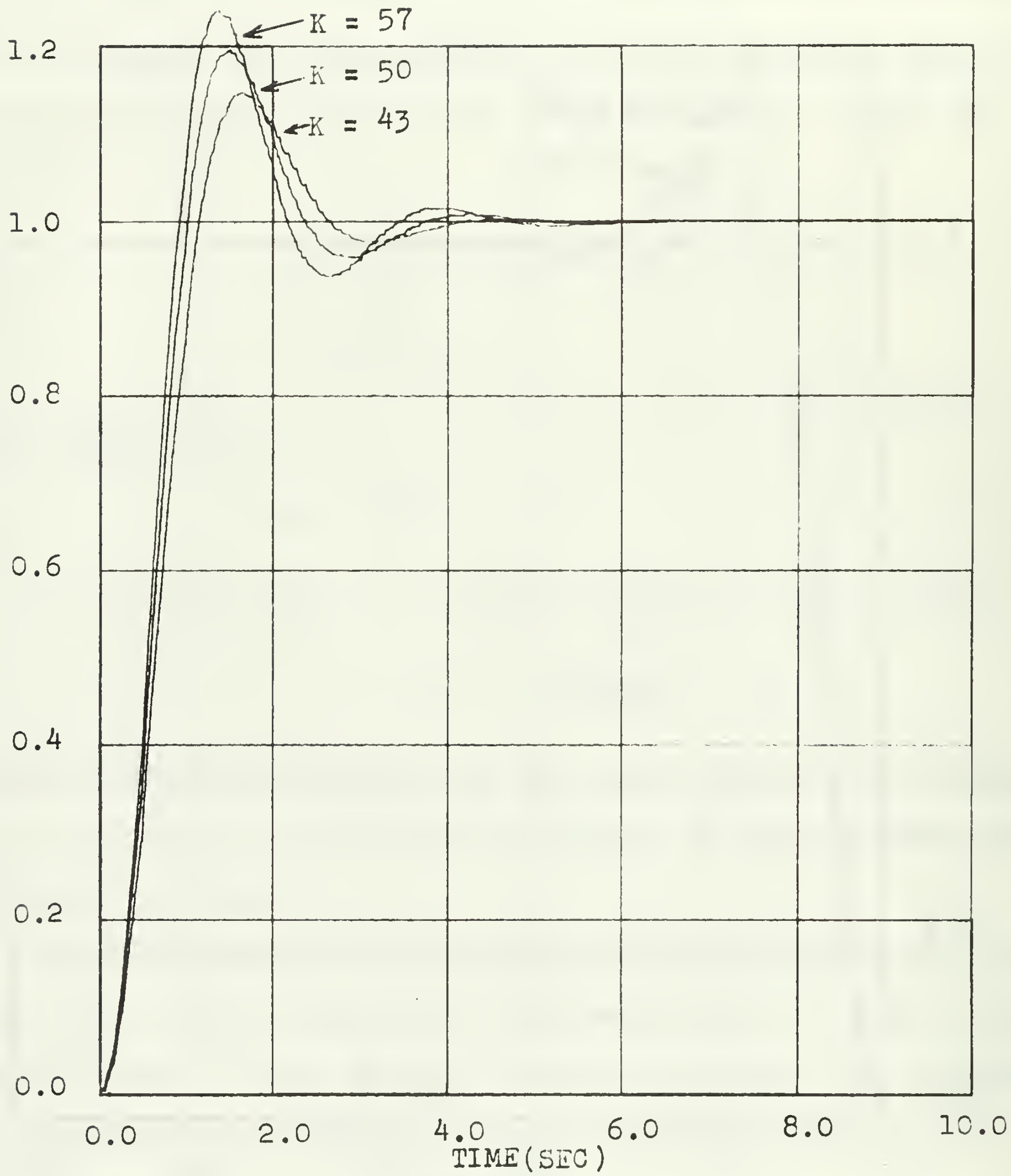


Fig.2.5. Gain tolerance of 14% for overshoot of ± 0.05.

BASIC SERVO SENSITIVITY COEFFICIENTS

TIME	OUT	SENSK	SENSF
0.0	0.0	0.0	-0.0
1.0000E-01	6.5182E-03	6.5160E-03	-2.2129E-06
2.0000E-01	4.1385E-02	4.1258E-02	-1.2685E-04
3.0000E-01	1.1165E-01	1.1054E-01	-1.1099E-03
4.0000E-01	2.1292E-01	2.0812E-01	-4.7931E-03
5.0000E-01	3.3623E-01	3.2212E-01	-1.4109E-02
6.0000E-01	4.7147E-01	4.3885E-01	-3.2619E-02
7.0000E-01	6.0911E-01	5.4526E-01	-6.3846E-02
8.0000E-01	7.4105E-01	6.3042E-01	-1.1063E-01
9.0000E-01	8.6100E-01	6.8640E-01	-1.7460E-01
1.0000E 00	9.6456E-01	7.0863E-01	-2.5593E-01
1.1000E 00	1.0491E 00	6.9590E-01	-3.5322E-01
1.2000E 00	1.1137E 00	6.5005E-01	-4.6363E-01
1.3000E 00	1.1586E 00	5.7542E-01	-5.8319E-01
1.4000E 00	1.1853E 00	4.7816E-01	-7.0712E-01
1.5000E 00	1.1959E 00	3.6558E-01	-8.3030E-01
1.6000E 00	1.1931E 00	2.4539E-01	-9.4767E-01
1.7000E 00	1.1797E 00	1.2510E-01	-1.0546E 00
1.8000E 00	1.1587E 00	1.1496E-02	-1.1472E 00
1.9000E 00	1.1328E 00	-8.9767E-02	-1.2226E 00
2.0000E 00	1.1046E 00	-1.7441E-01	-1.2790E 00
2.1000E 00	1.0760E 00	-2.3964E-01	-1.3157E 00
2.2000E 00	1.0489E 00	-2.8418E-01	-1.3331E 00
2.3000E 00	1.0245E 00	-3.0811E-01	-1.3326E 00
2.4000E 00	1.0036E 00	-3.1273E-01	-1.3163E 00
2.5000E 00	9.8672E-01	-3.0032E-01	-1.2870E 00
2.6000E 00	9.7404E-01	-2.7383E-01	-1.2479E 00
2.7000E 00	9.6544E-01	-2.3663E-01	-1.2021E 00
2.8000E 00	9.6056E-01	-1.9225E-01	-1.1528E 00
2.9000E 00	9.5895E-01	-1.4414E-01	-1.1031E 00
3.0000E 00	9.6002E-01	-9.5469E-02	-1.0555E 00
3.1000E 00	9.6318E-01	-4.8996E-02	-1.0122E 00
3.2000E 00	9.6782E-01	-6.9429E-03	-9.7476E-01
3.3000E 00	9.7336E-01	2.9047E-02	-9.4432E-01
3.4000E 00	9.7932E-01	5.7914E-02	-9.2140E-01
3.5000E 00	9.8525E-01	7.9165E-02	-9.0609E-01
3.6000E 00	9.9083E-01	9.2808E-02	-8.9802E-01
3.7000E 00	9.9581E-01	9.9281E-02	-8.9653E-01
3.8000E 00	1.0000E 00	9.9353E-02	-9.0066E-01
3.9000E 00	1.0034E 00	9.4037E-02	-9.0933E-01
4.0000E 00	1.0058E 00	8.4486E-02	-9.2136E-01
4.1000E 00	1.0075E 00	7.1906E-02	-9.3558E-01
4.2000E 00	1.0084E 00	5.7483E-02	-9.5088E-01
4.3000E 00	1.0086E 00	4.2310E-02	-9.6627E-01
4.4000E 00	1.0083E 00	2.7350E-02	-9.8091E-01
4.5000E 00	1.0075E 00	1.3390E-02	-9.9414E-01
4.6000E 00	1.0065E 00	1.0386E-03	-1.0055E 00
4.7000E 00	1.0053E 00	-9.2916E-03	-1.0146E 00
4.8000E 00	1.0041E 00	-1.7368E-02	-1.0214E 00
4.9000E 00	1.0028E 00	-2.3124E-02	-1.0260E 00
5.0000E 00	1.0017E 00	-2.6629E-02	-1.0283E 00
5.1000E 00	1.0007E 00	-2.8075E-02	-1.0288E 00
5.2000E 00	9.9984E-01	-2.7730E-02	-1.0276E 00
5.3000E 00	9.9918E-01	-2.5919E-02	-1.0251E 00
5.4000E 00	9.9869E-01	-2.2996E-02	-1.0217E 00

Table 2.1. Computer output data for unity feedback system.

BASIC SERVO SENSITIVITY COEFFICIENTS

TIME	OUT	SENSK	SENSF
5.5000E 00	9.9838E-01	-1.9314E-02	-1.0177E 00
5.6000E 00	9.9823E-01	-1.5208E-02	-1.0134E 00
5.7000E 00	9.9821E-01	-1.0980E-02	-1.0092E 00
5.8000E 00	9.9829E-01	-6.8853E-03	-1.0052E 00
5.9000E 00	9.9846E-01	-3.1281E-03	-1.0016E 00
6.0000E 00	9.9868E-01	1.4061E-04	-9.9854E-01
6.1000E 00	9.9893E-01	2.8257E-03	-9.9611E-01
6.2000E 00	9.9920E-01	4.8800E-03	-9.9432E-01
6.3000E 00	9.9945E-01	6.3002E-03	-9.9315E-01
6.4000E 00	9.9969E-01	7.1194E-03	-9.9257E-01
6.5000E 00	9.9989E-01	7.3986E-03	-9.9249E-01
6.6000E 00	1.0001E 00	7.2171E-03	-9.9284E-01
6.7000E 00	1.0002E 00	6.6686E-03	-9.9352E-01
6.8000E 00	1.0003E 00	5.8481E-03	-9.9444E-01
6.9000E 00	1.0003E 00	4.8504E-03	-9.9549E-01
7.0000E 00	1.0004E 00	3.7631E-03	-9.9661E-01
7.1000E 00	1.0004E 00	2.6627E-03	-9.9771E-01
7.2000E 00	1.0003E 00	1.6125E-03	-9.9873E-01
7.3000E 00	1.0003E 00	6.6310E-04	-9.9965E-01
7.4000E 00	1.0003E 00	-1.5068E-04	-1.0004E 00
7.5000E 00	1.0002E 00	-8.0872E-04	-1.0010E 00
7.6000E 00	1.0002E 00	-1.3018E-03	-1.0015E 00
7.7000E 00	1.0001E 00	-1.6327E-03	-1.0017E 00
7.8000E 00	1.0001E 00	-1.8120E-03	-1.0019E 00
7.9000E 00	1.0000E 00	-1.8587E-03	-1.0019E 00
8.0000E 00	9.9998E-01	-1.7906E-03	-1.0018E 00
8.1000E 00	9.9995E-01	-1.6360E-03	-1.0016E 00
8.2000E 00	9.9994E-01	-1.4186E-03	-1.0014E 00
8.3000E 00	9.9992E-01	-1.1616E-03	-1.0011E 00
8.4000E 00	9.9992E-01	-8.8680E-04	-1.0008E 00
8.5000E 00	9.9992E-01	-6.1244E-04	-1.0005E 00
8.6000E 00	9.9993E-01	-3.5483E-04	-1.0003E 00
8.7000E 00	9.9993E-01	-1.2463E-04	-1.0001E 00
8.8000E 00	9.9994E-01	6.9797E-05	-9.9987E-01
8.9000E 00	9.9995E-01	2.2489E-04	-9.9973E-01
9.0000E 00	9.9997E-01	3.3873E-04	-9.9963E-01
9.1000E 00	9.9998E-01	4.1258E-04	-9.9956E-01
9.2000E 00	9.9999E-01	4.4990E-04	-9.9954E-01
9.3000E 00	9.9999E-01	4.5526E-04	-9.9954E-01
9.4000E 00	1.0000E 00	4.3404E-04	-9.9957E-01
9.5000E 00	1.0000E 00	3.9238E-04	-9.9961E-01
9.6000E 00	1.0000E 00	3.3724E-04	-9.9967E-01
9.7000E 00	1.0000E 00	2.7323E-04	-9.9974E-01
9.8000E 00	1.0000E 00	2.0635E-04	-9.9981E-01
9.9000E 00	1.0000E 00	1.4067E-04	-9.9987E-01
1.0000E 01	1.0000E 00	8.0466E-05	-9.9993E-01

Table 2.1. (Continued)

TIME	K=50 OUT	K=55 OUT	K=45 OUT	K=36 OUT
0.0	0.0	0.0	0.0	0.0
1.0000E-01	6.5182E-03	7.1698E-03	5.8666E-03	4.6935E-03
2.0000E-01	4.1385E-02	4.5510E-02	3.7258E-02	2.9823E-02
3.0000E-01	1.1165E-01	1.2269E-01	1.0058E-01	8.0610E-02
4.0000E-01	2.1292E-01	2.3368E-01	1.9206E-01	1.5427E-01
5.0000E-01	3.3623E-01	3.6830E-01	3.0387E-01	2.4494E-01
6.0000E-01	4.7147E-01	5.1503E-01	4.2726E-01	3.4607E-01
7.0000E-01	6.0911E-01	6.6301E-01	5.5396E-01	4.5154E-01
8.0000E-01	7.4105E-01	8.0304E-01	6.7694E-01	5.5613E-01
9.0000E-01	8.6100E-01	9.2801E-01	7.9071E-01	6.5572E-01
1.0000E 00	9.6456E-01	1.0331E 00	8.9133E-01	7.4727E-01
1.1000E 00	1.0491E 00	1.1156E 00	9.7634E-01	8.2873E-01
1.2000E 00	1.1137E 00	1.1747E 00	1.0446E 00	8.9895E-01
1.3000E 00	1.1586E 00	1.2114E 00	1.0961E 00	9.5749E-01
1.4000E 00	1.1853E 00	1.2276E 00	1.1317E 00	1.0045E 00
1.5000E 00	1.1959E 00	1.2264E 00	1.1529E 00	1.0407E 00
1.6000E 00	1.1931E 00	1.2112E 00	1.1617E 00	1.0668E 00
1.7000E 00	1.1797E 00	1.1857E 00	1.1601E 00	1.0842E 00
1.8000E 00	1.1587E 00	1.1536E 00	1.1505E 00	1.0941E 00
1.9000E 00	1.1328E 00	1.1180E 00	1.1350E 00	1.0977E 00
2.0000E 00	1.1046E 00	1.0820E 00	1.1158E 00	1.0963E 00
2.1000E 00	1.0760E 00	1.0478E 00	1.0946E 00	1.0911E 00
2.2000E 00	1.0489E 00	1.0173E 00	1.0729E 00	1.0832E 00
2.3000E 00	1.0245E 00	9.9169E-01	1.0520E 00	1.0736E 00
2.4000E 00	1.0036E 00	9.7157E-01	1.0328E 00	1.0630E 00
2.5000E 00	9.8672E-01	9.5715E-01	1.0159E 00	1.0522E 00
2.6000E 00	9.7404E-01	9.4823E-01	1.0018E 00	1.0416E 00
2.7000E 00	9.6544E-01	9.4430E-01	9.9052E-01	1.0316E 00
2.8000E 00	9.6056E-01	9.4463E-01	9.8215E-01	1.0226E 00
2.9000E 00	9.5895E-01	9.4837E-01	9.7648E-01	1.0146E 00
3.0000E 00	9.6002E-01	9.5462E-01	9.7324E-01	1.0078E 00
3.1000E 00	9.6318E-01	9.6250E-01	9.7208E-01	1.0023E 00
3.2000E 00	9.6782E-01	9.7120E-01	9.7262E-01	9.9790E-01
3.3000E 00	9.7336E-01	9.8001E-01	9.7449E-01	9.9459E-01
3.4000E 00	9.7932E-01	9.8836E-01	9.7729E-01	9.9227E-01
3.5000E 00	9.8525E-01	9.9581E-01	9.8068E-01	9.9081E-01
3.6000E 00	9.9083E-01	1.0021E 00	9.8437E-01	9.9007E-01
3.7000E 00	9.9581E-01	1.0070E 00	9.8810E-01	9.8992E-01
3.8000E 00	1.0000E 00	1.0105E 00	9.9166E-01	9.9023E-01
3.9000E 00	1.0034E 00	1.0127E 00	9.9490E-01	9.9089E-01
4.0000E 00	1.0058E 00	1.0136E 00	9.9773E-01	9.9179E-01
4.1000E 00	1.0075E 00	1.0135E 00	1.0001E 00	9.9283E-01
4.2000E 00	1.0084E 00	1.0126E 00	1.0019E 00	9.9394E-01
4.3000E 00	1.0086E 00	1.0111E 00	1.0033E 00	9.9506E-01
4.4000E 00	1.0083E 00	1.0092E 00	1.0042E 00	9.9613E-01
4.5000E 00	1.0075E 00	1.0070E 00	1.0047E 00	9.9712E-01
4.6000E 00	1.0065E 00	1.0049E 00	1.0048E 00	9.9801E-01
4.7000E 00	1.0053E 00	1.0028E 00	1.0047E 00	9.9878E-01
4.8000E 00	1.0041E 00	1.0010E 00	1.0043E 00	9.9943E-01
4.9000E 00	1.0028E 00	9.9948E-01	1.0038E 00	9.9996E-01
5.0000E 00	1.0017E 00	9.9828E-01	1.0032E 00	1.0004E 00
5.1000E 00	1.0007E 00	9.9742E-01	1.0026E 00	1.0007E 00
5.2000E 00	9.9984E-01	9.9689E-01	1.0019E 00	1.0009E 00
5.3000E 00	9.9918E-01	9.9666E-01	1.0013E 00	1.0010E 00
5.4000E 00	9.9869E-01	9.9669E-01	1.0008E 00	1.0010E 00

Table 2.2. Computer output data for K = 55, 50, 45, and 36.

TIME	K=50 OUT	K=55 OUT	K=45 OUT	K=36 OUT
5.5000E 00	9.9838E-01	9.9691E-01	1.0003E 00	1.0010E 00
5.6000E 00	9.9823E-01	9.9729E-01	9.9992E-01	1.0010E 00
5.7000E 00	9.9821E-01	9.9776E-01	9.9962E-01	1.0009E 00
5.8000E 00	9.9829E-01	9.9828E-01	9.9940E-01	1.0008E 00
5.9000E 00	9.9846E-01	9.9881E-01	9.9926E-01	1.0007E 00
6.0000E 00	9.9868E-01	9.9931E-01	9.9919E-01	1.0006E 00
6.1000E 00	9.9893E-01	9.9975E-01	9.9917E-01	1.0005E 00
6.2000E 00	9.9920E-01	1.0001E 00	9.9920E-01	1.0004E 00
6.3000E 00	9.9945E-01	1.0004E 00	9.9927E-01	1.0003E 00
6.4000E 00	9.9969E-01	1.0006E 00	9.9936E-01	1.0002E 00
6.5000E 00	9.9989E-01	1.0008E 00	9.9946E-01	1.0001E 00
6.6000E 00	1.0001E 00	1.0008E 00	9.9957E-01	1.0000E 00
6.7000E 00	1.0002E 00	1.0008E 00	9.9968E-01	9.9998E-01
6.8000E 00	1.0003E 00	1.0008E 00	9.9978E-01	9.9995E-01
6.9000E 00	1.0003E 00	1.0007E 00	9.9988E-01	9.9992E-01
7.0000E 00	1.0004E 00	1.0005E 00	9.9996E-01	9.9990E-01
7.1000E 00	1.0004E 00	1.0004E 00	1.0000E 00	9.9989E-01
7.2000E 00	1.0003E 00	1.0003E 00	1.0001E 00	9.9989E-01
7.3000E 00	1.0003E 00	1.0002E 00	1.0001E 00	9.9989E-01
7.4000E 00	1.0003E 00	1.0001E 00	1.0001E 00	9.9990E-01
7.5000E 00	1.0002E 00	9.9997E-01	1.0001E 00	9.9991E-01
7.6000E 00	1.0002E 00	9.9989E-01	1.0001E 00	9.9992E-01
7.7000E 00	1.0001E 00	9.9984E-01	1.0001E 00	9.9993E-01
7.8000E 00	1.0001E 00	9.9981E-01	1.0001E 00	9.9994E-01
7.9000E 00	1.0000E 00	9.9980E-01	1.0001E 00	9.9995E-01
8.0000E 00	9.9998E-01	9.9980E-01	1.0001E 00	9.9996E-01
8.1000E 00	9.9995E-01	9.9981E-01	1.0001E 00	9.9997E-01
8.2000E 00	9.9994E-01	9.9984E-01	1.0000E 00	9.9998E-01
8.3000E 00	9.9992E-01	9.9986E-01	1.0000E 00	9.9999E-01
8.4000E 00	9.9992E-01	9.9989E-01	1.0000E 00	9.9999E-01
8.5000E 00	9.9992E-01	9.9993E-01	1.0000E 00	1.0000E 00
8.6000E 00	9.9993E-01	9.9996E-01	9.9999E-01	1.0000E 00
8.7000E 00	9.9993E-01	9.9998E-01	9.9998E-01	1.0000E 00
8.8000E 00	9.9994E-01	1.0000E 00	9.9998E-01	1.0000E 00
8.9000E 00	9.9995E-01	1.0000E 00	9.9997E-01	1.0000E 00
9.0000E 00	9.9997E-01	1.0000E 00	9.9997E-01	1.0000E 00
9.1000E 00	9.9998E-01	1.0000E 00	9.9997E-01	1.0000E 00
9.2000E 00	9.9999E-01	1.0000E 00	9.9997E-01	1.0000E 00
9.3000E 00	9.9999E-01	1.0000E 00	9.9998E-01	1.0000E 00
9.4000E 00	1.0000E 00	1.0000E 00	9.9998E-01	1.0000E 00
9.5000E 00	1.0000E 00	1.0000E 00	9.9998E-01	1.0000E 00
9.6000E 00	1.0000E 00	1.0000E 00	9.9999E-01	1.0000E 00
9.7000E 00	1.0000E 00	1.0000E 00	9.9999E-01	1.0000E 00
9.8000E 00	1.0000E 00	1.0000E 00	9.9999E-01	1.0000E 00
9.9000E 00	1.0000E 00	1.0000E 00	9.9999E-01	1.0000E 00
1.0000E 01	1.0000E 00	1.0000E 00	1.0000E 00	1.0000E 00

Table 2.2. (Continued)

If the design criteria is to restrict the peak overshoot to $\pm$ 0.05 then (2.9) gives

$$\Delta K/K = \frac{0.05}{0.366} \simeq 14\%$$

Figure 2.5 shows the results of the extreme values of K = 43 and K = 57 which give the tolerable limits of overshoot.

III. MULTILOOP CONTROL SYSTEM SENSITIVITY

A. INTRODUCTION

The Kokotovic [6] method of sensitivity analysis is expanded to investigate gain and time-contant variations in multiloop systems using a procedure similar to the one developed for the basic feedback control system. This procedure is developed using the block diagram of a regulated power generator which includes an amplifier-exciter, a series RC compensating circuit in the feedback loop, and a load as shown in Fig. 3.1.

B. GAIN-CONSTANT SENSITIVITY COEFFICIENTS

Terms are defined as follows:

$K_1G_1(s) \equiv$ Direct transfer function for the amplifier-exciter.

$K_2G_2(s) \equiv$ Transfer function for the generator.

$K_3G_3(s) \equiv$ Transfer function for the load.

$FH(s) \equiv$ Feedback compensator transfer function.

The basic equations are as follows:

$$\frac{C}{R}(s) = \frac{K_1G_1(s)K_2G_2(s)K_3G_3(s)}{1+K_1G_1(s)\,FH(s)+K_1G_1(s)K_2G_2(s)K_3G_3(s)} \qquad (3.1a)$$

$$\frac{E_2}{R}(s) = \frac{1}{1+K_1G_1(s)FH(s)+K_1G_1(s)K_2G_2(s)K_3G_3(s)} \qquad (3.1b)$$

$$\frac{E_1}{R}(s) = \frac{1+K_1G_1(s)FH(s)}{1+K_1G_1(s)FH(s)+K_1G_1(s)K_2G_2(s)K_3G_3(s)} \qquad (3.1c)$$

$$\frac{B_1}{R}(s) = \frac{K_1G_1(s)FH(s)}{1+K_1G_1(s)FH(s)+K_1G_1(s)K_2G_2(s)K_3G_3(s)} \qquad (3.1d)$$

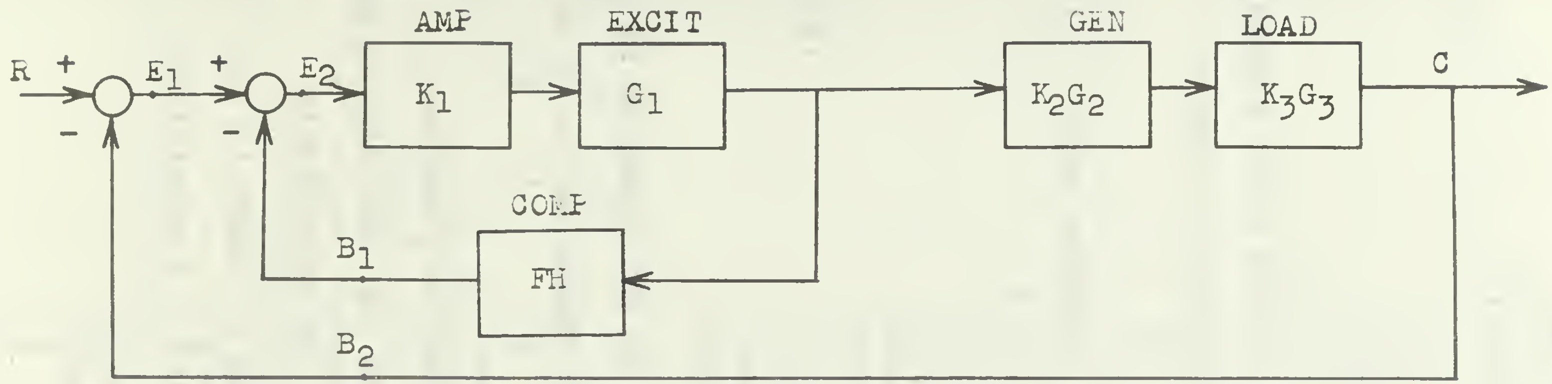
As before, capital letters denote the Laplace transform of the function described, except for K and F which are gain constants, and T which is a time constant.

To obtain the system sensitivity function apply (2.1) to (3.1a) to give

$$\begin{aligned} S_{K_1} &= \frac{RK_1G_1K_2G_2K_3G_3}{(1+K_1G_1FH+K_1G_1K_2G_2K_3G_3)^2} \\ &= \frac{C}{1+K_1G_1FH+K_1G_1K_2G_2K_3G_3} \end{aligned} \qquad (3.2)$$

This sensitivity function corresponds to the error signal, E_2, in the original multiloop system but with the controlled output, C, as the input to the system. This can be seen by comparing (3.2) to (3.1b) with C replacing R. As in the basic control system of Section II, the sensitivity model for gain parameters can be realized by cascading two identical systems and using the error signals in the sensitivity model as the desired gain sensitivity coefficients as shown in Fig. 3.2.

The load-gain sensitivity function is derived by applying (2.1) to (3.1a), yielding



$$K_1G_1 = \frac{K}{Ts+1} = \frac{40}{4s+1}$$

$$K_2G_2 = \frac{K}{Ts+1} = \frac{1}{2s+1}$$

$$K_3G_3 = \frac{K}{Ts+1} = \frac{1}{s+1}$$

$$FH = \frac{KTs}{Ts+1} = \frac{2s}{s+1}$$

Fig.3.1. Multiloop system.

$$S_{K_3} = \frac{RK_1G_1K_2G_2K_3G_3(1+K_1G_1FH)}{(1+K_1G_1FH+K_1G_1K_2G_2K_3G_3)^2}$$

$$= \frac{C(1+K_1G_1FH)}{1+K_1G_1FH+K_1G_1K_2G_2K_3G_3} \qquad (3.3)$$

This sensitivity function corresponds to the error signal, E_1, as can be seen by comparing (3.3) to (3.1c) with C replacing R as the input.

For the feedback-gain sensitivity function apply (2.1) to (3.1a), yielding

$$S_F = \frac{RK_1G_1K_2G_2K_3G_3(-K_1G_1FH)}{1+K_1G_1FH+K_1G_1K_2G_2K_3G_3)^2}$$

$$= \frac{C(-K_1G_1FH)}{1+K_1G_1FH+K_1G_1K_2G_2K_3G_3} \qquad (3.4)$$

This sensitivity function corresponds to the negative of the feedback signal, B_1, as can be seen by comparing (3.4) to (3.1d) with C replacing R as before.

By applying (2.1) to the system transfer function, sensitivity functions have been derived for the various gain parameters and the pickoff points for the sensitivity functions can be selected by inspection as shown in Fig. 3.2. No new components have been required and all sensitivity functions can be generated simultaneously.

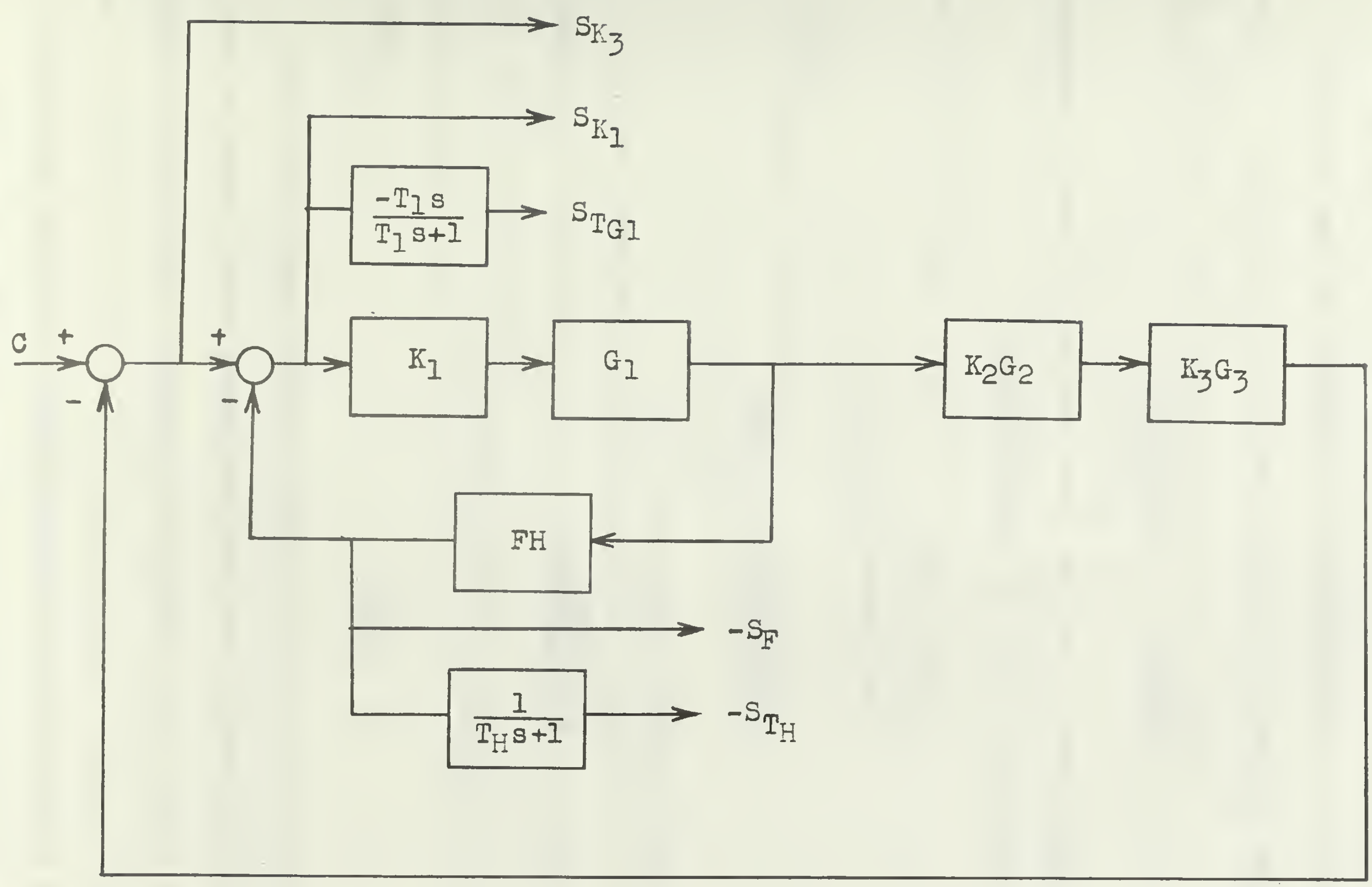


Fig.3.2. Multiloop sensitivity model.

C. TIME-CONSTANT SENSITIVITY COEFFICIENTS

Since time-constant variations can also affect the response, sensitivity coefficients for LR and CR network combinations can add helpful information to the analysis of a modeled system. For this derivation, let

$$\frac{C}{R} = \frac{KG}{1+KGFH} = \frac{KG(Ts+1)}{1+Ts(KGF+1)} \tag{3.5}$$

where $H = \frac{Ts}{Ts+1}$ and T is the time constant of an RC feedback circuit. Applying (2.1) to (3.5) yields

$$\begin{aligned} S_T &= \frac{-R(KG)^2 FTs}{[1+Ts(KGF+1)]^2} \\ &= \frac{C(-KGFTs)}{[1+Ts(KGF+1)]} \frac{1}{Ts+1} \end{aligned} \tag{3.6}$$

and the feedback-gain sensitivity is

$$S_F = \frac{C(-KGFTs)}{1+Ts(KGF+1)} \tag{3.7}$$

Therefore, by combining (3.5) and (3.6)

$$S_T = S_F \left(\frac{1}{Ts+1}\right) \tag{3.8}$$

This indicates that the RC time-constant sensitivity can be generated by using the feedback-gain sensitivity as the input to a block with elementary transfer function as shown in Fig. 3.2.

In the direct path there are time-constants associated with the generator and load inductance and resistance combinations. The effect of variations of these time-constants will add information to the analysis of the system. For this derivation, let

$$\frac{C}{R} = \frac{KG}{1+KGFH} = \frac{K}{1+Ts+KFH} \tag{3.9}$$

where $G = \frac{1}{Ts+1}$, and T is the time-constant of a direct path LR component. Applying (2.1) to (3.9) yields

$$\begin{aligned} S_T &= \frac{-RKTs}{(1+Ts+KFH)^2} \\ &= \frac{C(-Ts)}{(1+Ts+KFH)} \end{aligned} \tag{3.10}$$

and the direct-gain sensitivity is

$$S_K = \frac{C(Ts+1)}{(1+Ts+KFH)} \tag{3.11}$$

Therefore, by combining (3.10) and (3.11)

$$S_T = S_K \left(\frac{-Ts}{Ts+1}\right) \tag{3.12}$$

As with the feedback RC time-constant sensitivity, only one elementary block has to be added in series with the direct-gain sensitivity model to complete the sensitivity model as shown in Fig. 3.2.

D. APPLICATION

One of the basic problems of preliminary design analysis is to estimate the influence of each design parameter on the system response and decide how improvement of the system can be assured. Sensitivity coefficients can be helpful in solving this problem. To illustrate this procedure, a basic set of design parameters for a regulated generator is taken to be amplifier gain, exciter time-constant, load-gain,

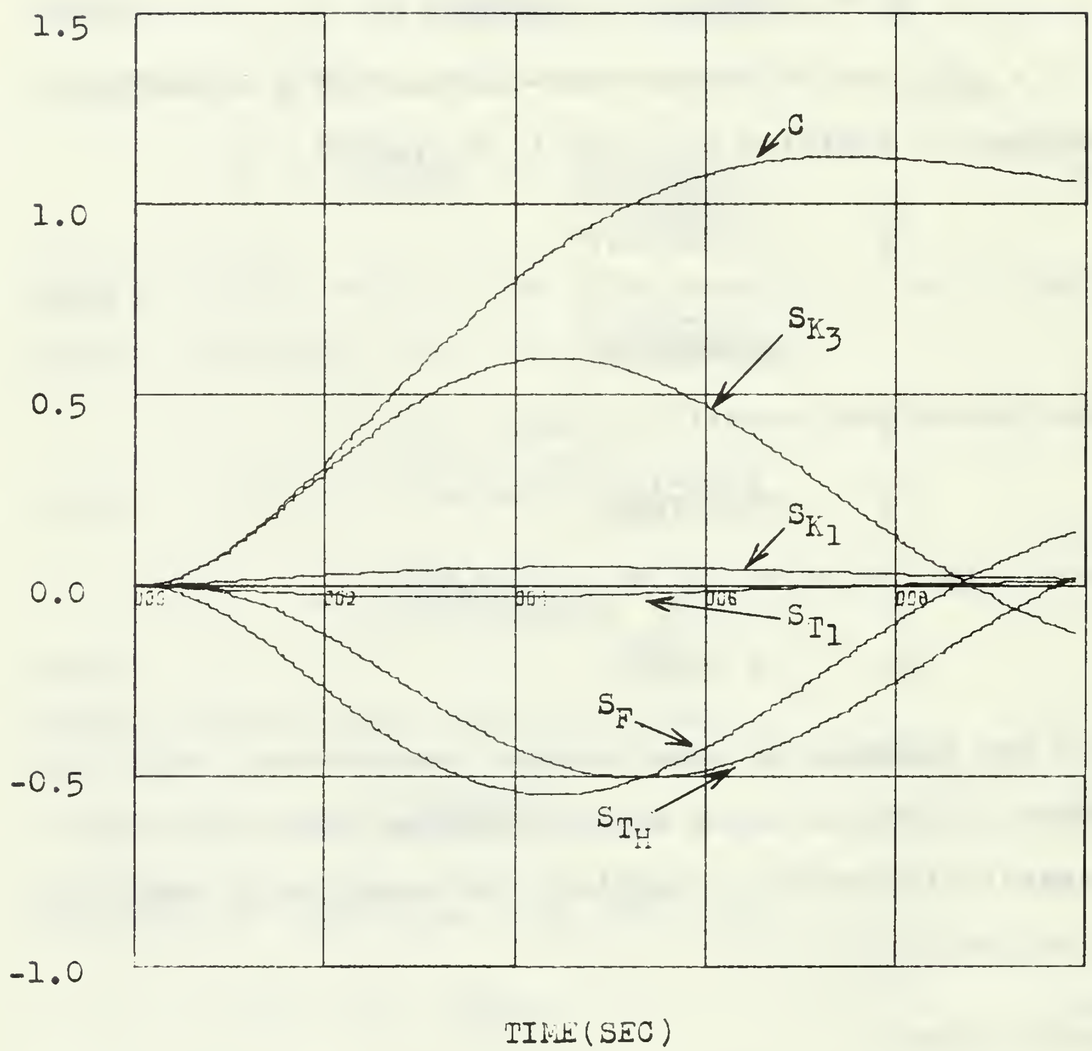


Fig.3.3. Multiloop system response and sensitivity functions.

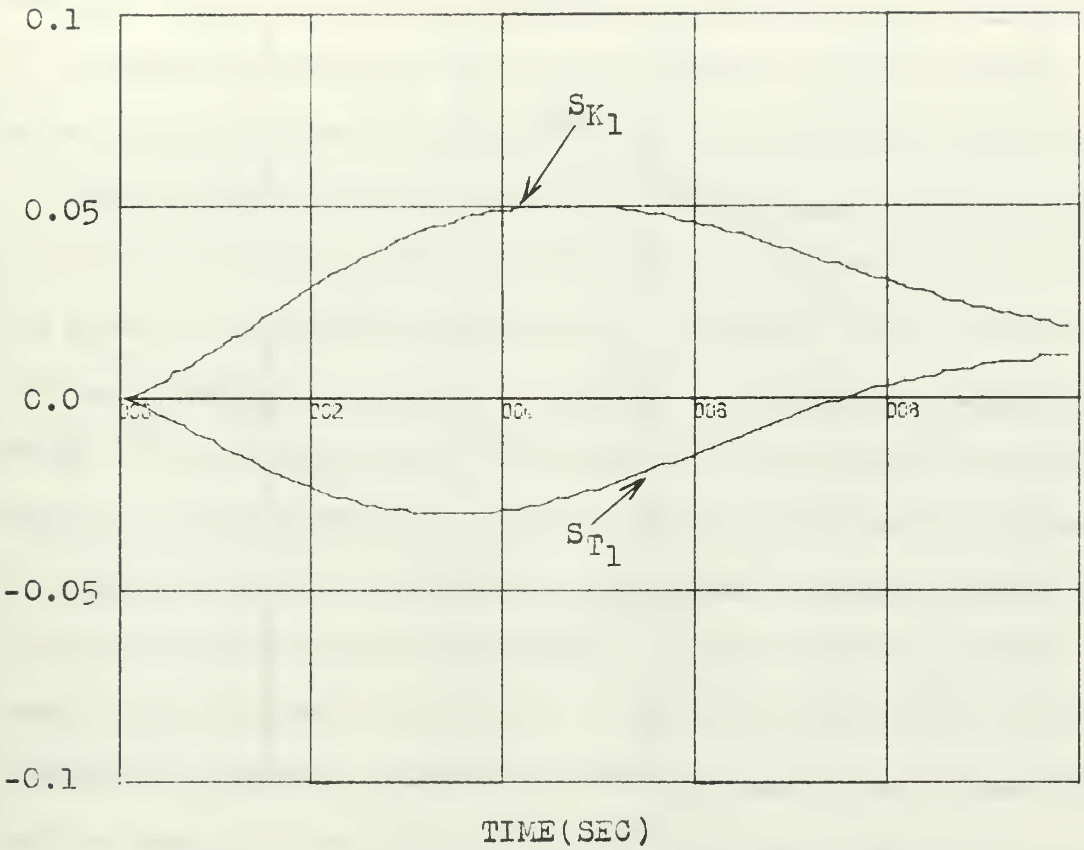


Fig.3.4. Amplifier-exciter sensitivity functions.

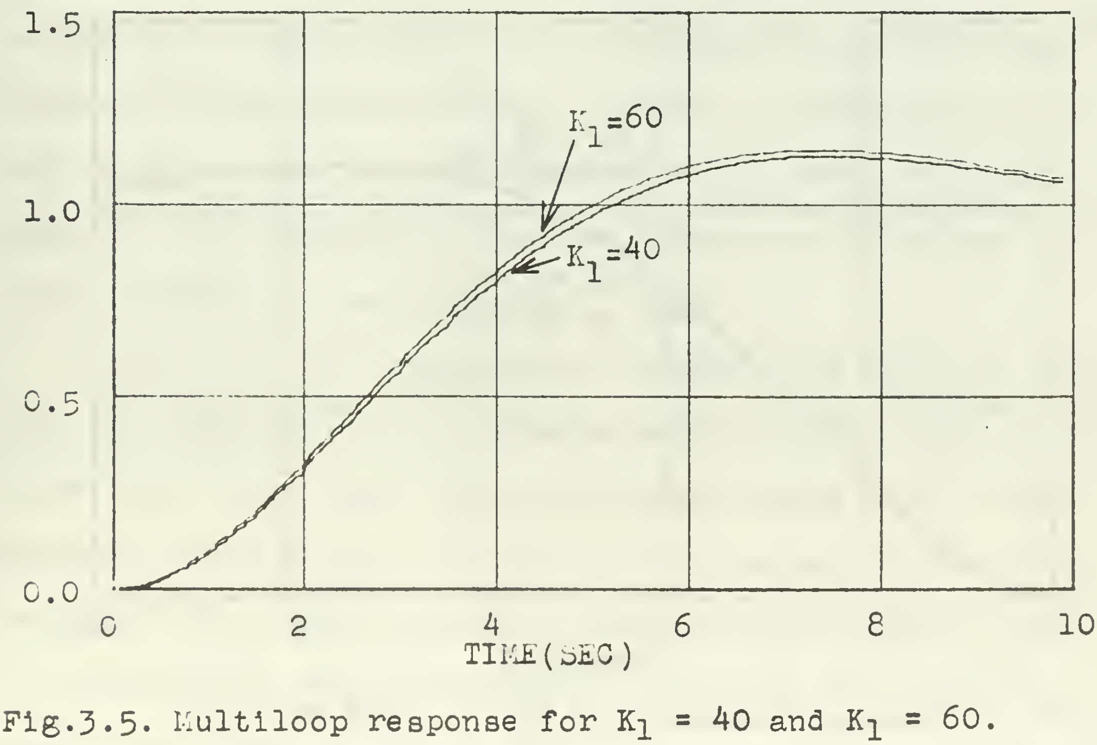


Fig.3.5. Multiloop response for K_1 = 40 and K_1 = 60.

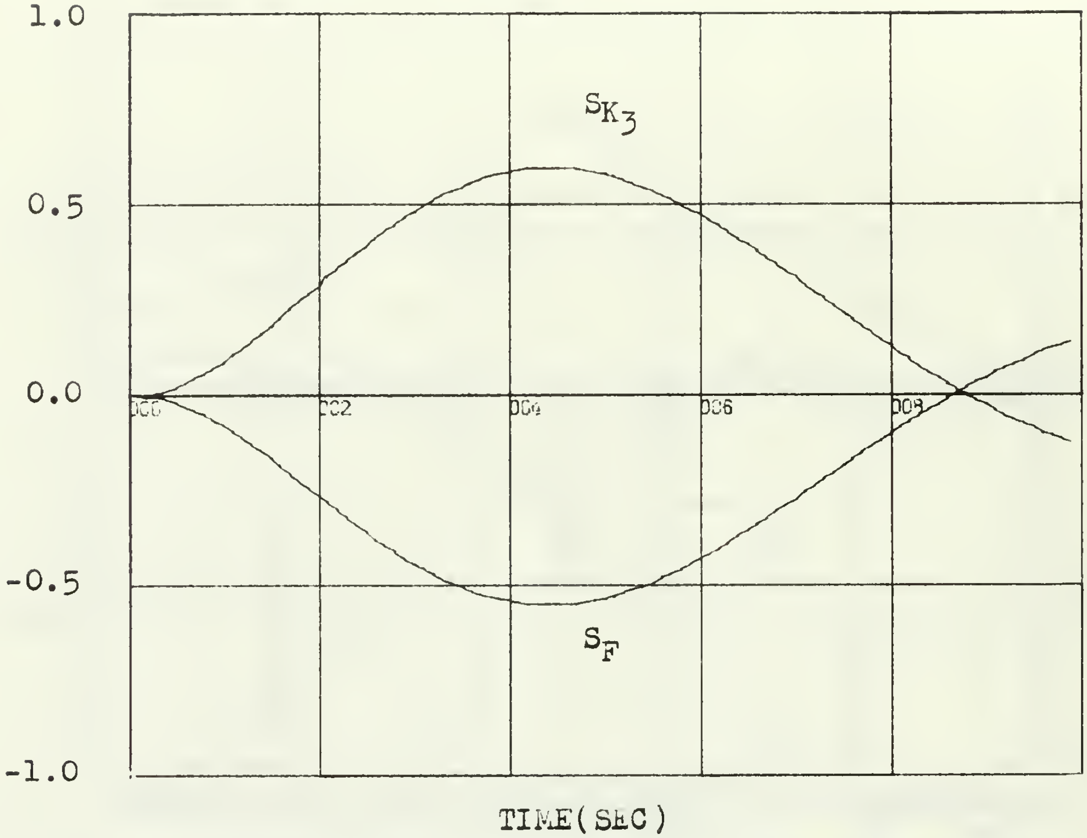


Fig.3.6. Load and feedback gain sensitivity functions.

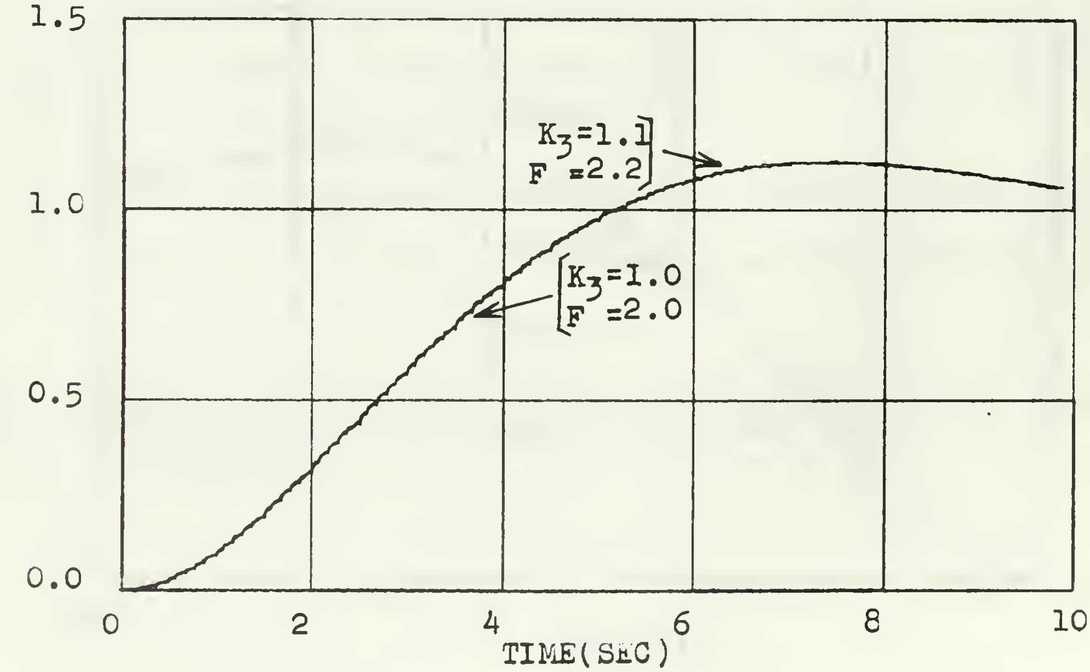


Fig.3.7. Multiloop response for K_3= 1.1 and F= 2.2.

feedback-gain, and a compensator time-constant. Figure 3.3 is a computer graph of the time response to a unit-step input and the five design-parameter sensitivity coefficients.

At a glance, it is apparent from Fig. 3.3 that K_1 and T_1 have relatively little influence on the response. Table 3.1 shows the maximum value of S_{K_1} is only 0.05 which is less than 10% the value of S_{K_3}. Figure 3.4 shows S_{K_1} and S_{T_1} where the scale has been increased by 10 to show the general shape of the curves. Figure 3.5 shows the time response for $K_1 = 40$ and $K_1 = 60$. Even for this 50% increase, there is little change in the performance. Since the response is very "insensitive" to K_1 and T_1, little improvement can be achieved by adjusting these parameters and their tolerances can be large in relation to the other parameters. It should be noted that the system is practically insensitive to small variations of resistance in the exciter because the time-constant and gain both either increase or decrease by the same amount. The sensitivities have the same relative amplitude but differ in sign and therefore the effect on the time response is canceled.

In Fig. 3.6 the sensitivity coefficients for the load-gain and feedback-gain constants are plotted. Table 3.2 shows that their magnitudes are almost equal but are 180 degrees out of phase. Figure 3.7 is a graph of the time response with their original values and the response for a 10% increase in both parameters. Although the response is sensitive to such changes, the plots are almost superimposed due to the one being the negative of the other. This could be favorable equalization if aging or temperature cause these gain parameters to vary by the same percentage.

Figure 3.3 shows that the feedback-gain and the compensator RC time-constant have almost the same sensitivity but the time-constant sensitivity function is delayed by one second.

Since fractional parameter variations are considered, the permissible percentage of a parameter change can be determined from the allowable system response variation.

It has been shown that any general form of a control system that can be transformed into a system of primary loops can be analyzed in the same fashion as developed in this section. The sensitivity to variations in direct- and feedback-gain constants are available as error or feedback signals in the sensitivity model and time-constant sensitivities require addition of elementary transfer functions as shown in Fig. 3.2.

TIME	C	S_{K_1}	S_{T_1}
0.0	0.0	0.0	0.0
1.0000E-01	5.2083E-04	3.2895E-04	-3.2650E-04
2.0000E-01	2.9189E-03	1.2793E-03	-1.2578E-03
3.0000E-01	7.4409E-03	2.4687E-03	-2.4017E-03
4.0000E-01	1.4014E-02	3.7582E-03	-3.6161E-03
5.0000E-01	2.2527E-02	5.1151E-03	-4.8670E-03
6.0000E-01	3.2866E-02	6.5283E-03	-6.1426E-03
7.0000E-01	4.4918E-02	7.9903E-03	-7.4349E-03
8.0000E-01	5.8570E-02	9.4944E-03	-8.7368E-03
9.0000E-01	7.3713E-02	1.1034E-02	-1.0042E-02
1.0000E 00	9.0238E-02	1.2603E-02	-1.1343E-02
1.1000E 00	1.0804E-01	1.4194E-02	-1.2635E-02
1.2000E 00	1.2701E-01	1.5801E-02	-1.3910E-02
1.3000E 00	1.4705E-01	1.7420E-02	-1.5165E-02
1.4000E 00	1.6807E-01	1.9043E-02	-1.6393E-02
1.5000E 00	1.8995E-01	2.0664E-02	-1.7590E-02
1.6000E 00	2.1262E-01	2.2280E-02	-1.8751E-02
1.7000E 00	2.3597E-01	2.3883E-02	-1.9872E-02
1.8000E 00	2.5993E-01	2.5470E-02	-2.0948E-02
1.9000E 00	2.8440E-01	2.7035E-02	-2.1977E-02
2.0000E 00	3.0930E-01	2.8574E-02	-2.2954E-02
2.1000E 00	3.3455E-01	3.0082E-02	-2.3877E-02
2.2000E 00	3.6007E-01	3.1556E-02	-2.4742E-02
2.3000E 00	3.8580E-01	3.2991E-02	-2.5549E-02
2.4000E 00	4.1166E-01	3.4385E-02	-2.6294E-02
2.5000E 00	4.3759E-01	3.5733E-02	-2.6976E-02
2.6000E 00	4.6351E-01	3.7032E-02	-2.7594E-02
2.7000E 00	4.8937E-01	3.8281E-02	-2.8146E-02
2.8000E 00	5.1512E-01	3.9476E-02	-2.8631E-02
2.9000E 00	5.4069E-01	4.0615E-02	-2.9048E-02
3.0000E 00	5.6603E-01	4.1696E-02	-2.9399E-02
3.1000E 00	5.9111E-01	4.2717E-02	-2.9681E-02
3.2000E 00	6.1586E-01	4.3677E-02	-2.9896E-02
3.3000E 00	6.4025E-01	4.4574E-02	-3.0044E-02
3.4000E 00	6.6424E-01	4.5408E-02	-3.0126E-02
3.5000E 00	6.8779E-01	4.6178E-02	-3.0142E-02
3.6000E 00	7.1086E-01	4.6881E-02	-3.0092E-02
3.7000E 00	7.3344E-01	4.7520E-02	-2.9980E-02
3.8000E 00	7.5548E-01	4.8093E-02	-2.9805E-02
3.9000E 00	7.7696E-01	4.8600E-02	-2.9570E-02
4.0000E 00	7.9786E-01	4.9042E-02	-2.9276E-02
4.1000E 00	8.1816E-01	4.9419E-02	-2.8925E-02
4.2000E 00	8.3784E-01	4.9731E-02	-2.8520E-02
4.3000E 00	8.5688E-01	4.9980E-02	-2.8062E-02
4.4000E 00	8.7528E-01	5.0168E-02	-2.7553E-02
4.5000E 00	8.9301E-01	5.0293E-02	-2.6997E-02
4.6000E 00	9.1006E-01	5.0357E-02	-2.6393E-02
4.7000E 00	9.2644E-01	5.0362E-02	-2.5746E-02
4.8000E 00	9.4214E-01	5.0309E-02	-2.5059E-02
4.9000E 00	9.5715E-01	5.0203E-02	-2.4335E-02
5.0000E 00	9.7147E-01	5.0039E-02	-2.3572E-02
5.1000E 00	9.8510E-01	4.9825E-02	-2.2779E-02
5.2000E 00	9.9804E-01	4.9561E-02	-2.1955E-02
5.3000E 00	1.0103E 00	4.9247E-02	-2.1103E-02
5.4000E 00	1.0219E 00	4.8887E-02	-2.0226E-02

Table 3.1a. Computer output data for C, S_{K_1}, and S_{T_1}.

TIME	C	S_{K_1}	S_{T_1}
5.5000E 00	1.0328E 00	4.8484E-02	-1.9329E-02
5.6000E 00	1.0430E 00	4.8035E-02	-1.8408E-02
5.7000E 00	1.0526E 00	4.7550E-02	-1.7475E-02
5.8000E 00	1.0615E 00	4.7024E-02	-1.6523E-02
5.9000E 00	1.0698E 00	4.6467E-02	-1.5566E-02
6.0000E 00	1.0775E 00	4.5875E-02	-1.4597E-02
6.1000E 00	1.0845E 00	4.5251E-02	-1.3620E-02
6.2000E 00	1.0910E 00	4.4601E-02	-1.2641E-02
6.3000E 00	1.0969E 00	4.3923E-02	-1.1660E-02
6.4000E 00	1.1021E 00	4.3223E-02	-1.0681E-02
6.5000E 00	1.1069E 00	4.2501E-02	-9.7037E-03
6.6000E 00	1.1111E 00	4.1759E-02	-8.7310E-03
6.7000E 00	1.1148E 00	4.0999E-02	-7.7655E-03
6.8000E 00	1.1179E 00	4.0227E-02	-6.8106E-03
6.9000E 00	1.1206E 00	3.9443E-02	-5.8688E-03
7.0000E 00	1.1228E 00	3.8649E-02	-4.9392E-03
7.1000E 00	1.1246E 00	3.7846E-02	-4.0244E-03
7.2000E 00	1.1259E 00	3.7040E-02	-3.1285E-03
7.3000E 00	1.1268E 00	3.6225E-02	-2.2473E-03
7.4000E 00	1.1273E 00	3.5413E-02	-1.3897E-03
7.5000E 00	1.1274E 00	3.4599E-02	-5.5103E-04
7.6000E 00	1.1271E 00	3.3787E-02	2.6421E-04
7.7000E 00	1.1266E 00	3.2978E-02	1.0564E-03
7.8000E 00	1.1256E 00	3.2178E-02	1.8213E-03
7.9000E 00	1.1244E 00	3.1382E-02	2.5623E-03
8.0000E 00	1.1229E 00	3.0596E-02	3.2745E-03
8.1000E 00	1.1211E 00	2.9821E-02	3.9590E-03
8.2000E 00	1.1191E 00	2.9058E-02	4.6156E-03
8.3000E 00	1.1168E 00	2.8305E-02	5.2448E-03
8.4000E 00	1.1143E 00	2.7569E-02	5.8424E-03
8.5000E 00	1.1116E 00	2.6848E-02	6.4102E-03
8.6000E 00	1.1087E 00	2.6145E-02	6.9460E-03
8.7000E 00	1.1056E 00	2.5457E-02	7.4535E-03
8.8000E 00	1.1024E 00	2.4791E-02	7.9278E-03
8.9000E 00	1.0991E 00	2.4144E-02	8.3706E-03
9.0000E 00	1.0956E 00	2.3515E-02	8.7855E-03
9.1000E 00	1.0920E 00	2.2910E-02	9.1657E-03
9.2000E 00	1.0883E 00	2.2327E-02	9.5148E-03
9.3000E 00	1.0845E 00	2.1766E-02	9.8346E-03
9.4000E 00	1.0806E 00	2.1228E-02	1.0123E-02
9.5000E 00	1.0767E 00	2.0713E-02	1.0382E-02
9.6000E 00	1.0728E 00	2.0222E-02	1.0610E-02
9.7000E 00	1.0688E 00	1.9754E-02	1.0810E-02
9.8000E 00	1.0647E 00	1.9313E-02	1.0979E-02
9.9000E 00	1.0607E 00	1.8897E-02	1.1118E-02
1.0000E 01	1.0567E 00	1.8503E-02	1.1233E-02
1.0100E 01	1.0527E 00	1.8134E-02	1.1320E-02
1.0200E 01	1.0486E 00	1.7792E-02	1.1378E-02
1.0300E 01	1.0446E 00	1.7472E-02	1.1413E-02
1.0400E 01	1.0407E 00	1.7179E-02	1.1421E-02
1.0500E 01	1.0368E 00	1.6908E-02	1.1407E-02
1.0600E 01	1.0329E 00	1.6664E-02	1.1366E-02
1.0700E 01	1.0291E 00	1.6441E-02	1.1305E-02
1.0800E 01	1.0253E 00	1.6244E-02	1.1221E-02
1.0900E 01	1.0216E 00	1.6069E-02	1.1116E-02

Table 3.1a. (Continued)

TIME	C	S_{K_3}	S_F	S_{T_H}
0.0	0.0	-0.0	0.0	0.0
1.0000E-01	5.2081E-04	5.2079E-04	-1.9186E-04	-4.2725E-06
2.0000E-01	2.9185E-03	2.9177E-03	-1.6383E-03	-7.8408E-05
3.0000E-01	7.4408E-03	7.4349E-03	-4.9661E-03	-3.7232E-04
4.0000E-01	1.4014E-02	1.3990E-02	-1.0232E-02	-1.0491E-03
5.0000E-01	2.2527E-02	2.2462E-02	-1.7346E-02	-2.2530E-03
6.0000E-01	3.2866E-02	3.2718E-02	-2.6190E-02	-4.1039E-03
7.0000E-01	4.4918E-02	4.4627E-02	-3.6636E-02	-6.6987E-03
8.0000E-01	5.8570E-02	5.8053E-02	-4.8559E-02	-1.0113E-02
9.0000E-01	7.3713E-02	7.2863E-02	-6.1829E-02	-1.4404E-02
1.0000E 00	9.0238E-02	8.8919E-02	-7.6317E-02	-1.9608E-02
1.1000E 00	1.0804E-01	1.0609E-01	-9.1894E-02	-2.5750E-02
1.2000E 00	1.2701E-01	1.2424E-01	-1.0843E-01	-3.2838E-02
1.3000E 00	1.4705E-01	1.4323E-01	-1.2581E-01	-4.0866E-02
1.4000E 00	1.6807E-01	1.6293E-01	-1.4389E-01	-4.9819E-02
1.5000E 00	1.8995E-01	1.8323E-01	-1.6256E-01	-5.9671E-02
1.6000E 00	2.1262E-01	2.0398E-01	-1.8170E-01	-7.0385E-02
1.7000E 00	2.3597E-01	2.2507E-01	-2.0119E-01	-8.1918E-02
1.8000E 00	2.5993E-01	2.4638E-01	-2.2091E-01	-9.4222E-02
1.9000E 00	2.8440E-01	2.6780E-01	-2.4076E-01	-1.0724E-01
2.0000E 00	3.0930E-01	2.8921E-01	-2.6063E-01	-1.2091E-01
2.1000E 00	3.3455E-01	3.1050E-01	-2.8042E-01	-1.3516E-01
2.2000E 00	3.6007E-01	3.3158E-01	-3.0002E-01	-1.4993E-01
2.3000E 00	3.8580E-01	3.5235E-01	-3.1935E-01	-1.6516E-01
2.4000E 00	4.1166E-01	3.7271E-01	-3.3832E-01	-1.8075E-01
2.5000E 00	4.3759E-01	3.9257E-01	-3.5684E-01	-1.9664E-01
2.6000E 00	4.6351E-01	4.1186E-01	-3.7483E-01	-2.1276E-01
2.7000E 00	4.8937E-01	4.3051E-01	-3.9222E-01	-2.2903E-01
2.8000E 00	5.1512E-01	4.4842E-01	-4.0895E-01	-2.4538E-01
2.9000E 00	5.4069E-01	4.6555E-01	-4.2494E-01	-2.6172E-01
3.0000E 00	5.6603E-01	4.8183E-01	-4.4014E-01	-2.7800E-01
3.1000E 00	5.9111E-01	4.9721E-01	-4.5449E-01	-2.9413E-01
3.2000E 00	6.1586E-01	5.1162E-01	-4.6795E-01	-3.1005E-01
3.3000E 00	6.4025E-01	5.2505E-01	-4.8047E-01	-3.2569E-01
3.4000E 00	6.6424E-01	5.3743E-01	-4.9202E-01	-3.4098E-01
3.5000E 00	6.8779E-01	5.4874E-01	-5.0256E-01	-3.5587E-01
3.6000E 00	7.1086E-01	5.5895E-01	-5.1206E-01	-3.7030E-01
3.7000E 00	7.3344E-01	5.6803E-01	-5.2051E-01	-3.8421E-01
3.8000E 00	7.5548E-01	5.7597E-01	-5.2788E-01	-3.9754E-01
3.9000E 00	7.7696E-01	5.8275E-01	-5.3415E-01	-4.1026E-01
4.0000E 00	7.9786E-01	5.8837E-01	-5.3933E-01	-4.2231E-01
4.1000E 00	8.1816E-01	5.9281E-01	-5.4339E-01	-4.3365E-01
4.2000E 00	8.3784E-01	5.9608E-01	-5.4635E-01	-4.4425E-01
4.3000E 00	8.5688E-01	5.9819E-01	-5.4821E-01	-4.5406E-01
4.4000E 00	8.7528E-01	5.9913E-01	-5.4896E-01	-4.6307E-01
4.5000E 00	8.9301E-01	5.9892E-01	-5.4863E-01	-4.7123E-01
4.6000E 00	9.1006E-01	5.9758E-01	-5.4722E-01	-4.7854E-01
4.7000E 00	9.2644E-01	5.9513E-01	-5.4476E-01	-4.8496E-01
4.8000E 00	9.4214E-01	5.9158E-01	-5.4127E-01	-4.9049E-01
4.9000E 00	9.5715E-01	5.8696E-01	-5.3675E-01	-4.9511E-01
5.0000E 00	9.7147E-01	5.8130E-01	-5.3126E-01	-4.9882E-01
5.1000E 00	9.8510E-01	5.7463E-01	-5.2480E-01	-5.0160E-01
5.2000E 00	9.9804E-01	5.6698E-01	-5.1742E-01	-5.0346E-01
5.3000E 00	1.0103E 00	5.5839E-01	-5.0915E-01	-5.0439E-01
5.4000E 00	1.0219E 00	5.4890E-01	-5.0001E-01	-5.0441E-01

Table 3.1b. Computer output data for C, S_{K_3}, S_F, and S_{T_H}.

TIME	C	S_{K_3}	S_F	S_{T_H}
5.5000E 00	1.0328E 00	5.3854E-01	-4.9006E-01	-5.0352E-01
5.6000E 00	1.0430E 00	5.2736E-01	-4.7932E-01	-5.0172E-01
5.7000E 00	1.0526E 00	5.1539E-01	-4.6784E-01	-4.9904E-01
5.8000E 00	1.0615E 00	5.0269E-01	-4.5566E-01	-4.9549E-01
5.9000E 00	1.0698E 00	4.8929E-01	-4.4282E-01	-4.9108E-01
6.0000E 00	1.0775E 00	4.7525E-01	-4.2937E-01	-4.8584E-01
6.1000E 00	1.0845E 00	4.6060E-01	-4.1535E-01	-4.7980E-01
6.2000E 00	1.0910E 00	4.4540E-01	-4.0079E-01	-4.7296E-01
6.3000E 00	1.0969E 00	4.2969E-01	-3.8576E-01	-4.6537E-01
6.4000E 00	1.1021E 00	4.1352E-01	-3.7029E-01	-4.5705E-01
6.5000E 00	1.1069E 00	3.9694E-01	-3.5443E-01	-4.4803E-01
6.6000E 00	1.1111E 00	3.7999E-01	-3.3823E-01	-4.3834E-01
6.7000E 00	1.1148E 00	3.6273E-01	-3.2173E-01	-4.2802E-01
6.8000E 00	1.1179E 00	3.4520E-01	-3.0497E-01	-4.1710E-01
6.9000E 00	1.1206E 00	3.2744E-01	-2.8800E-01	-4.0561E-01
7.0000E 00	1.1228E 00	3.0951E-01	-2.7086E-01	-3.9359E-01
7.1000E 00	1.1246E 00	2.9144E-01	-2.5360E-01	-3.8107E-01
7.2000E 00	1.1259E 00	2.7329E-01	-2.3625E-01	-3.6810E-01
7.3000E 00	1.1268E 00	2.5509E-01	-2.1887E-01	-3.5471E-01
7.4000E 00	1.1273E 00	2.3690E-01	-2.0148E-01	-3.4094E-01
7.5000E 00	1.1274E 00	2.1874E-01	-1.8414E-01	-3.2683E-01
7.6000E 00	1.1271E 00	2.0066E-01	-1.6687E-01	-3.1242E-01
7.7000E 00	1.1266E 00	1.8270E-01	-1.4972E-01	-2.9774E-01
7.8000E 00	1.1256E 00	1.6490E-01	-1.3272E-01	-2.8283E-01
7.9000E 00	1.1244E 00	1.4728E-01	-1.1590E-01	-2.6773E-01
8.0000E 00	1.1229E 00	1.2990E-01	-9.9302E-02	-2.5247E-01
8.1000E 00	1.1211E 00	1.1277E-01	-8.2952E-02	-2.3710E-01
8.2000E 00	1.1191E 00	9.5940E-02	-6.6882E-02	-2.2165E-01
8.3000E 00	1.1168E 00	7.9426E-02	-5.1121E-02	-2.0616E-01
8.4000E 00	1.1143E 00	6.3260E-02	-3.5691E-02	-1.9066E-01
8.5000E 00	1.1116E 00	4.7470E-02	-2.0622E-02	-1.7518E-01
8.6000E 00	1.1087E 00	3.2081E-02	-5.9357E-03	-1.5976E-01
8.7000E 00	1.1056E 00	1.7114E-02	8.3437E-03	-1.4442E-01
8.8000E 00	1.1024E 00	2.5921E-03	2.2199E-02	-1.2921E-01
8.9000E 00	1.0991E 00	-1.1465E-02	3.5609E-02	-1.1415E-01
9.0000E 00	1.0956E 00	-2.5040E-02	4.8554E-02	-9.9269E-02
9.1000E 00	1.0920E 00	-3.8116E-02	6.1026E-02	-8.4595E-02
9.2000E 00	1.0883E 00	-5.0678E-02	7.3006E-02	-7.0153E-02
9.3000E 00	1.0845E 00	-6.2716E-02	8.4481E-02	-5.5971E-02
9.4000E 00	1.0806E 00	-7.4216E-02	9.5444E-02	-4.2070E-02
9.5000E 00	1.0767E 00	-8.5169E-02	1.0588E-01	-2.8474E-02
9.6000E 00	1.0728E 00	-9.5567E-02	1.1579E-01	-1.5205E-02
9.7000E 00	1.0688E 00	-1.0540E-01	1.2516E-01	-2.2815E-03
9.8000E 00	1.0647E 00	-1.1468E-01	1.3399E-01	1.0278E-02
9.9000E 00	1.0607E 00	-1.2338E-01	1.4227E-01	2.2456E-02
1.0000E 01	1.0567E 00	-1.3151E-01	1.5001E-01	3.4237E-02
1.0100E 01	1.0527E 00	-1.3907E-01	1.5720E-01	4.5607E-02
1.0200E 01	1.0486E 00	-1.4606E-01	1.6385E-01	5.6552E-02
1.0300E 01	1.0446E 00	-1.5247E-01	1.6995E-01	6.7062E-02
1.0400E 01	1.0407E 00	-1.5833E-01	1.7551E-01	7.7126E-02
1.0500E 01	1.0368E 00	-1.6362E-01	1.8053E-01	8.6736E-02
1.0600E 01	1.0329E 00	-1.6836E-01	1.8502E-01	9.5883E-02
1.0700E 01	1.0291E 00	-1.7255E-01	1.8899E-01	1.0456E-01
1.0800E 01	1.0253E 00	-1.7620E-01	1.9245E-01	1.1277E-01
1.0900E 01	1.0216E 00	-1.7932E-01	1.9539E-01	1.2050E-01

Table 3.1b. (Continued)

IV. EXAMPLES OF DIFFERENTIAL EQUATION SENSITIVITY ANALYSIS

A. INTRODUCTION

A general method of determining the sensitivity functions for the coefficients of a second-order differential equation is derived for a mass, spring, and damper system. The procedure is then applied to the study of a sixth-order differential equation, the solution of which is initially unstable. The coefficients are then adjusted to yield a stable solution using the results of sensitivity analysis.

B. SECOND-ORDER DIFFERENTIAL EQUATION SENSITIVITY TO COEFFICIENT VARIATIONS

The elementary mass-spring-damper system shown in Fig. 4.1 is described by the following differential equation

$$M\ddot{X} + B\dot{X} + KX = u(t) \tag{4.1}$$

Taking the Laplace transform and solving for X(s) gives

$$X(s) = \frac{U(s)}{Ms^2+Bs+K} \tag{4.2}$$

For convenience, let M = 1. Applying (2.1) to (4.2) yields

$$S_K = \frac{-K\ U(s)}{(s^2+Bs+K)^2} = X(s)\left(\frac{-K}{s^2+Bs+K}\right) \tag{4.3}$$

By comparing (4.3) to (4.2) it can be seen that the sensitivity model is a replica of the original system with the controlled output as the input to the model. The sensitivity coefficient is measured at the point corresponding to the feedback signal.

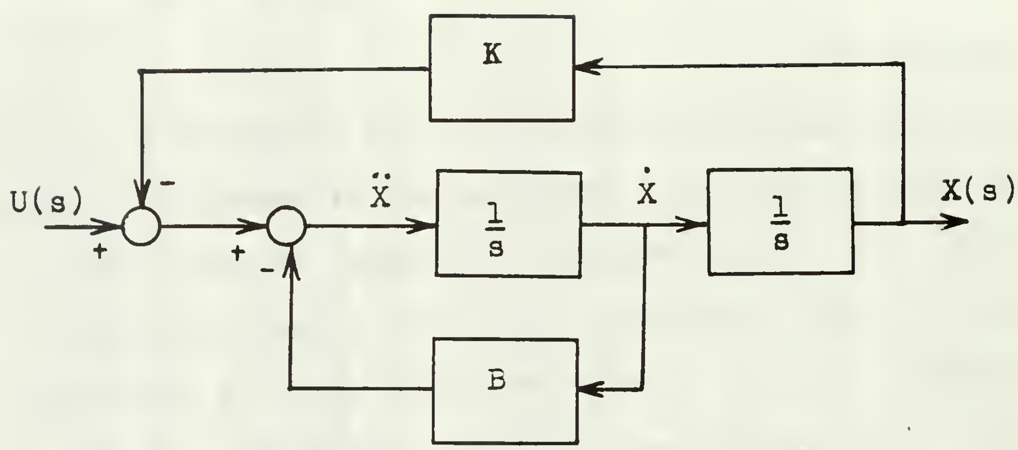


Fig.4.1. Block diagram of mass-spring-damper system.

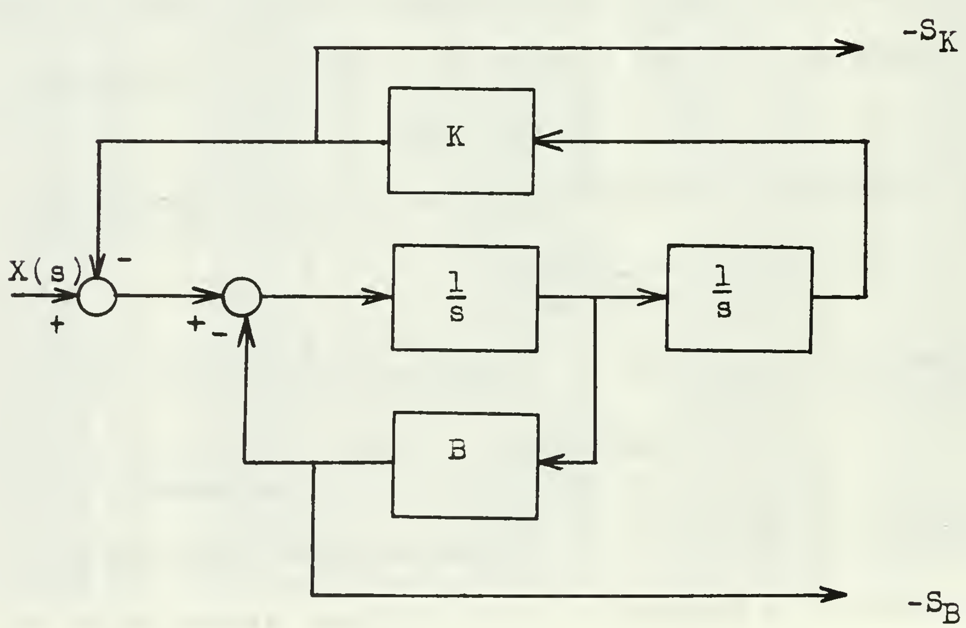


Fig.4.2. Second-order differential equation sensitivity model.

Similarly,

$$S_B = \frac{-Bs\ U(s)}{(s^2+Bs+K)^2} = X(s)\ \left(\frac{-Bs}{s^2+Bs+K}\right) \qquad (4.4)$$

Figure 4.2 is a block diagram of the complete sensitivity model which shows clearly that S_K and S_B are comparable to the feedback gain sensitivities as derived in Section II and III.

The solution of (4.1) for a unit-impulse input is

$$x(t) = \frac{1}{\sqrt{K - \frac{B^2}{4}}} e^{-\frac{B}{2}t} \sin \sqrt{K - \frac{B^2}{4}}\ t \qquad (4.5)$$

Figure 4.4 is the graph of x(t) and the sensitivity coefficients S_K and S_B for B = 0.6 and K = 0.6. The plot shows that the system is more sensitive to changes in K than B as verified by (4.5). Table 4.1 is the computer output data for χ, S_K, and S_B.

C. SIXTH-ORDER DIFFERENTIAL EQUATION SENSITIVITY TO COEFFICIENT VARIATIONS

Consider the differential equation

$$a\frac{d^6x}{dt^6} + b\frac{d^5x}{dt^5} + c\frac{d^4x}{dt^4} + d\frac{d^3x}{dt^3} + e\frac{d^2x}{dt^2} + f\frac{dx}{dt} + g\,x = u(t) \qquad (4.6)$$

where nominally a = 1, b = 3.4, c = 11.4, d = 9.4, e = 7.4, f = 0.575, g = 0.452. The solution when u(t) is a unit-step can be determined by DSL using only one block. For the sensitivity model the equation must be integrated in six steps to isolate each sensitivity coefficient. The procedure used for the second-degree equation can be expanded to give

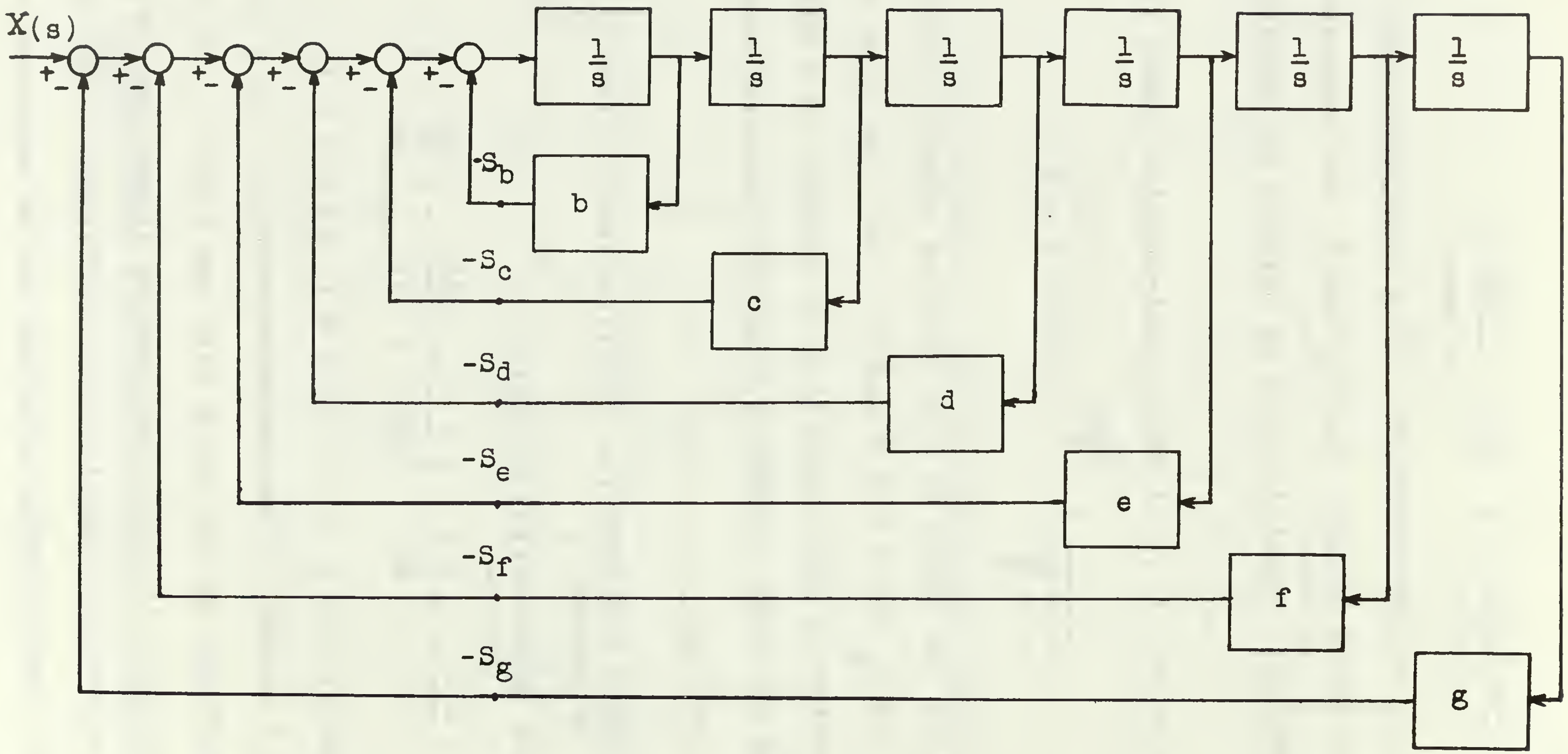


Fig.4.3. Sixth-order differential equation sensitivity model.

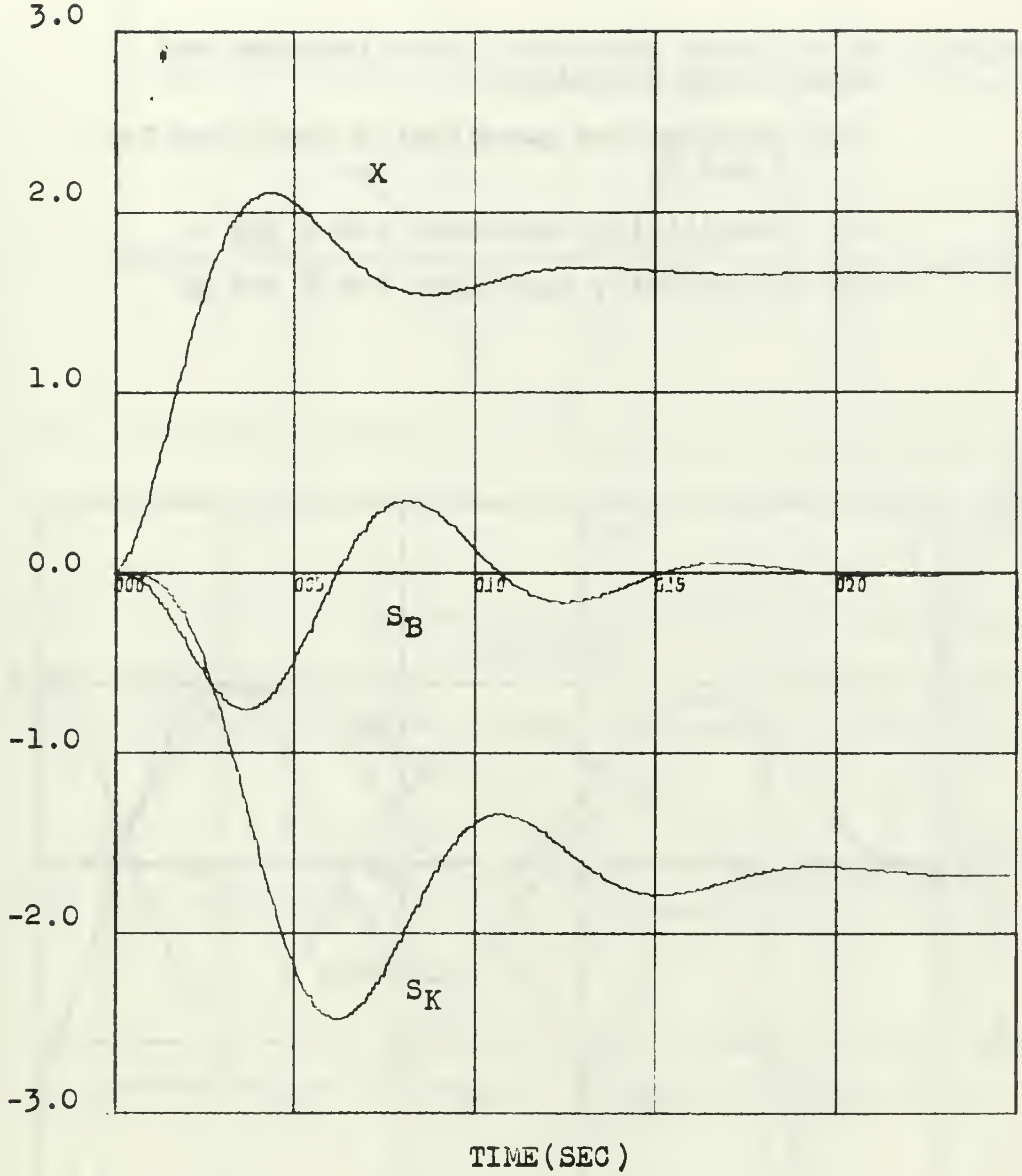


Fig.4.4. Second-order system response and sensitivity functions.

Fig.4.5. Sixth-order unstable system response and sensitivity functions.

(a). Response and sensitivity functions for d and f.

(b). Sensitivity functions for b and c.

(c). Sensitivity functions for e and g.

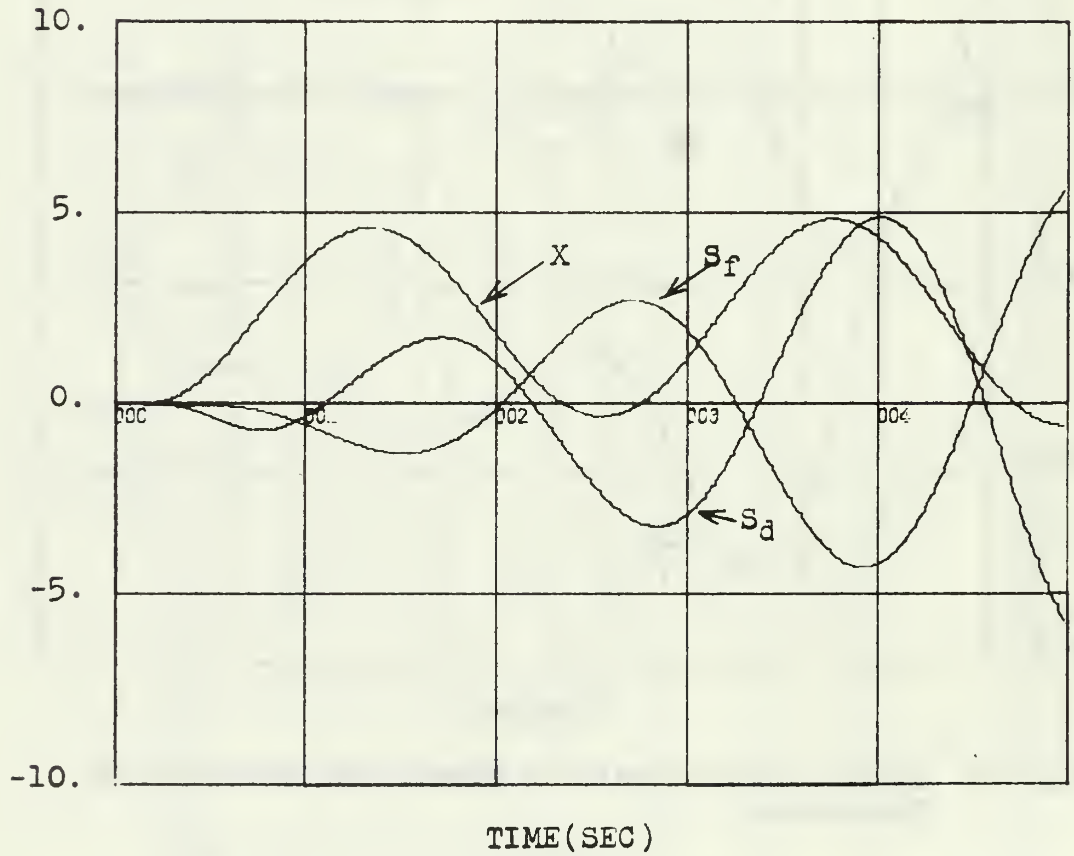


Fig.4.5(a). X, S_d, and S_f.

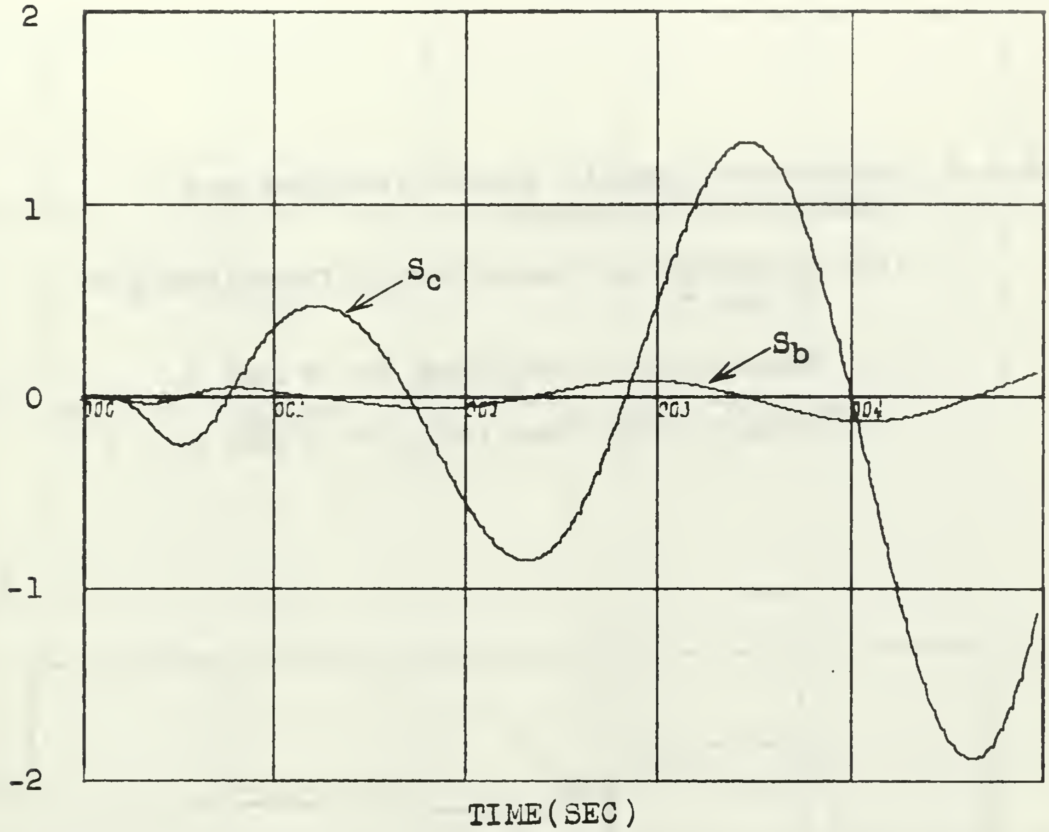


Fig.4.5(b). S_b and S_c.

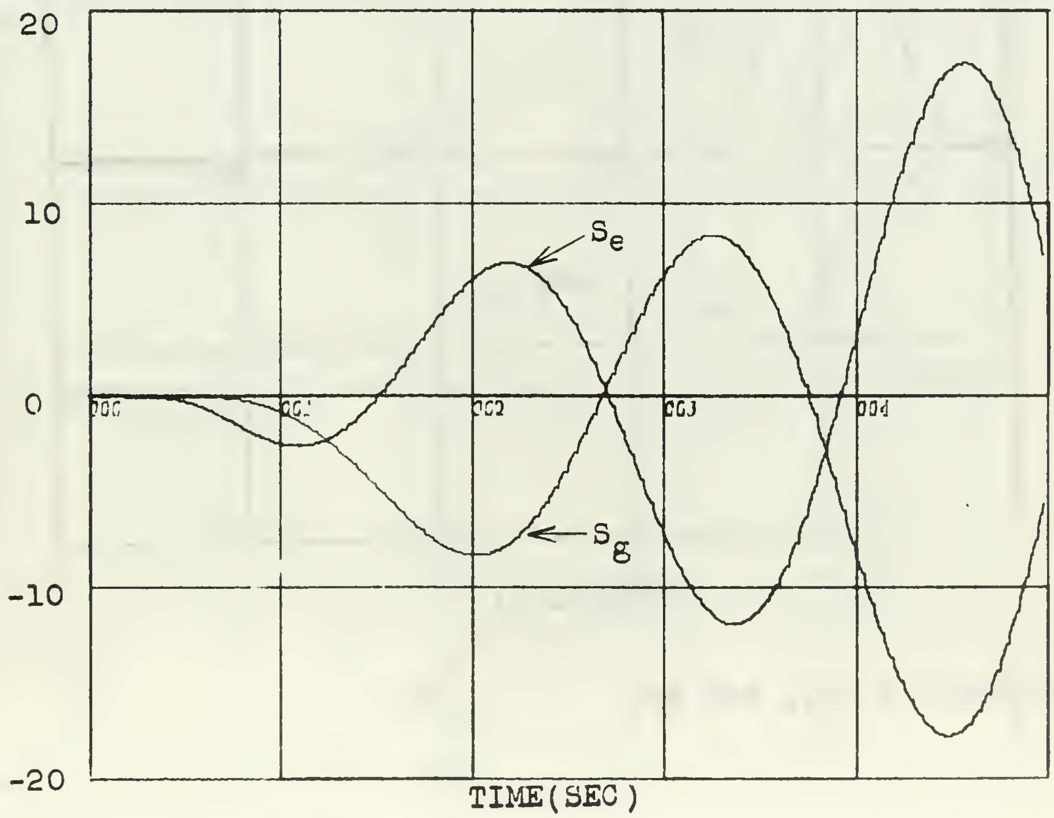


Fig.4.5(c). S_e and S_g.

Fig.4.6. Sixth-order stable system response and sensitivity functions.

(a). Response and sensitivity functions for d and f.

(b). Sensitivity functions for b and c.

(c). Sensitivity functions for e and g.

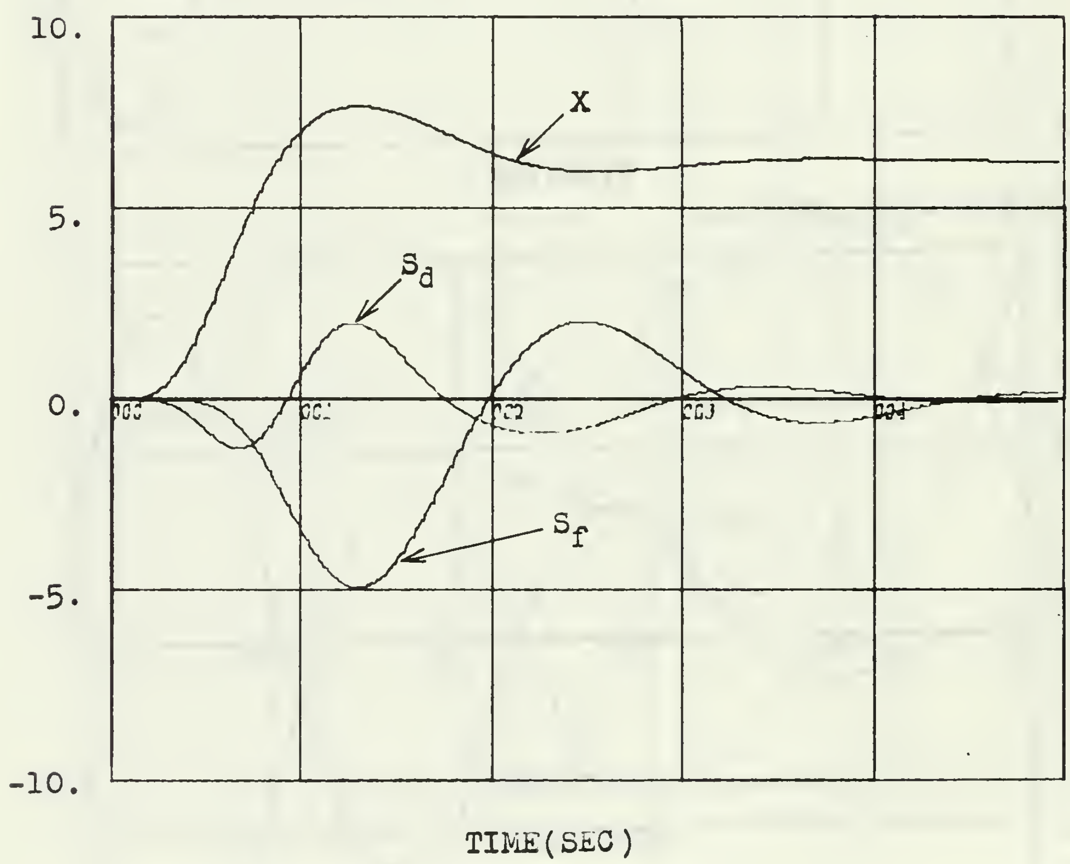


Fig.4.6(a). X, S_d, and S_f.

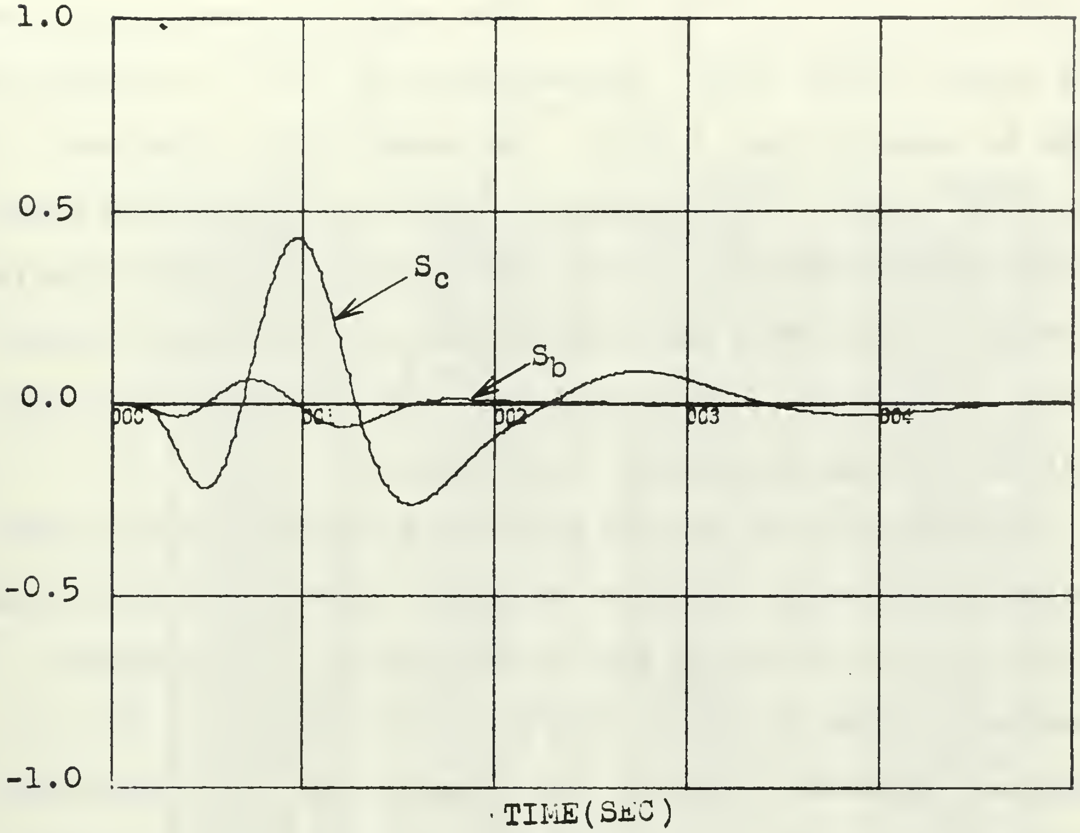


Fig.4.6(b). S_b and S_c.

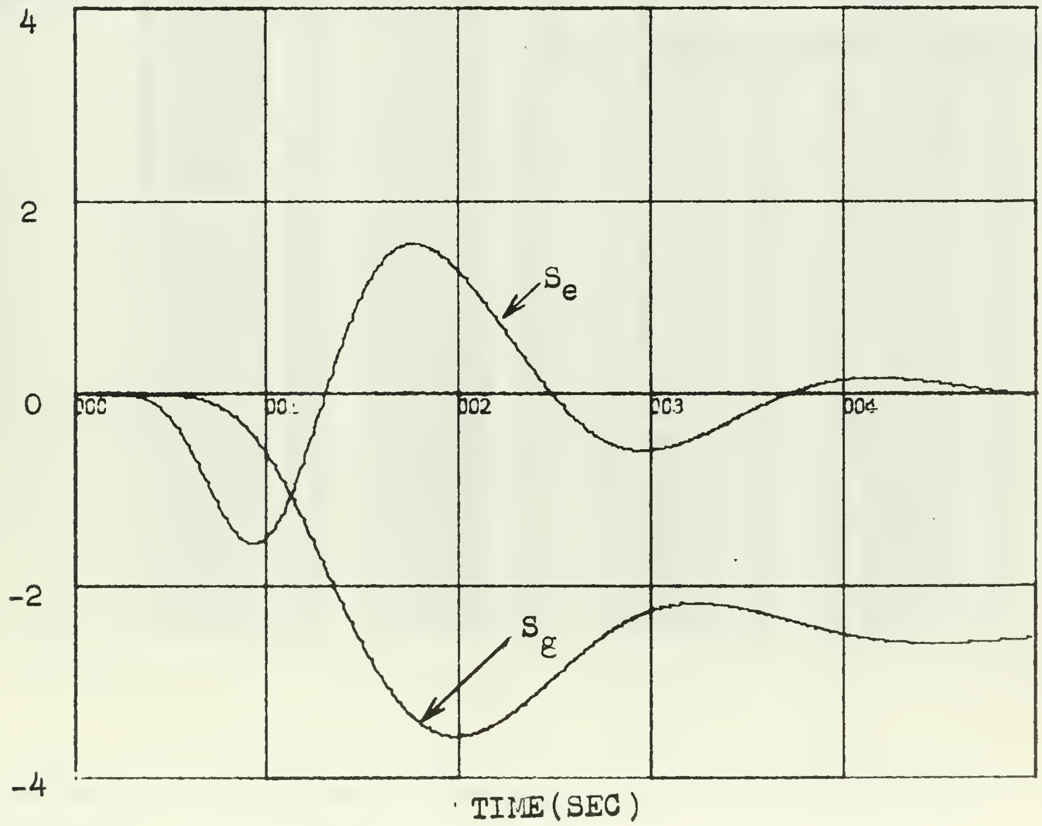


Fig.4.6(c). S_e and S_g.

a model similar to Fig. 4.2. The expanded sensitivity model is shown in Fig. 4.3. The solution of (4.6) is unstable as can be seen in Fig. 4.5(a). The sensitivity functions graphed in Fig. 4.5(b) indicate that the inner-loop feedback gain parameters b and c influence the output relatively little. Going from the coefficient of the highest order term to the lowest, the influence increases but it is obtained with an increasing time delay.

The problem of stabilizing the solution may be solved using sensitivity analysis by noting that S_f is out-of-phase with X. By increasing the coefficient f, the response becomes stable as shown in Fig. 4.6(a) for f = 1.8. Any further adjustment of the performance can be achieved by referring to the sensitivity functions for the stabilized system as shown in Fig. 4.6.

TIME	X	SENSB	SENSK
0.0	0.0	-0.0	-0.0
1.0000E-01	4.8990E-03	-9.6995E-05	-2.4401E-06
2.0000E-01	1.9185E-02	-7.5187E-04	-3.8079E-05
3.0000E-01	4.2229E-02	-2.4563E-03	-1.8789E-04
4.0000E-01	7.3386E-02	-5.6301E-03	-5.7840E-04
5.0000E-01	1.1200E-01	-1.0622E-02	-1.3746E-03
6.0000E-01	1.5742E-01	-1.7713E-02	-2.7729E-03
7.0000E-01	2.0898E-01	-2.7114E-02	-4.9943E-03
8.0000E-01	2.6601E-01	-3.8976E-02	-8.2778E-03
9.0000E-01	3.2788E-01	-5.3385E-02	-1.2874E-02
1.0000E 00	3.9394E-01	-7.0372E-02	-1.9041E-02
1.1000E 00	4.6356E-01	-8.9912E-02	-2.7034E-02
1.2000E 00	5.3613E-01	-1.1193E-01	-3.7106E-02
1.3000E 00	6.1106E-01	-1.3632E-01	-4.9500E-02
1.4000E 00	6.8776E-01	-1.6290E-01	-6.4443E-02
1.5000E 00	7.6570E-01	-1.9150E-01	-8.2148E-02
1.6000E 00	8.4434E-01	-2.2187E-01	-1.0280E-01
1.7000E 00	9.2319E-01	-2.5377E-01	-1.2657E-01
1.8000E 00	1.0018E 00	-2.8692E-01	-1.5360E-01
1.9000E 00	1.0797E 00	-3.2102E-01	-1.8399E-01
2.0000E 00	1.1564E 00	-3.5577E-01	-2.1783E-01
2.1000E 00	1.2317E 00	-3.9085E-01	-2.5516E-01
2.2000E 00	1.3051E 00	-4.2594E-01	-2.9600E-01
2.3000E 00	1.3763E 00	-4.6072E-01	-3.4034E-01
2.4000E 00	1.4451E 00	-4.9487E-01	-3.8812E-01
2.5000E 00	1.5112E 00	-5.2807E-01	-4.3928E-01
2.6000E 00	1.5743E 00	-5.6003E-01	-4.9370E-01
2.7000E 00	1.6343E 00	-5.9045E-01	-5.5123E-01
2.8000E 00	1.6909E 00	-6.1905E-01	-6.1173E-01
2.9000E 00	1.7442E 00	-6.4558E-01	-6.7498E-01
3.0000E 00	1.7939E 00	-6.6981E-01	-7.4077E-01
3.1000E 00	1.8399E 00	-6.9151E-01	-8.0885E-01
3.2000E 00	1.8823E 00	-7.1049E-01	-8.7898E-01
3.3000E 00	1.9209E 00	-7.2659E-01	-9.5086E-01
3.4000E 00	1.9558E 00	-7.3966E-01	-1.0242E 00
3.5000E 00	1.9870E 00	-7.4959E-01	-1.0987E 00
3.6000E 00	2.0145E 00	-7.5629E-01	-1.1740E 00
3.7000E 00	2.0384E 00	-7.5968E-01	-1.2498E 00
3.8000E 00	2.0587E 00	-7.5974E-01	-1.3258E 00
3.9000E 00	2.0756E 00	-7.5646E-01	-1.4017E 00
4.0000E 00	2.0891E 00	-7.4985E-01	-1.4770E 00
4.1000E 00	2.0994E 00	-7.3995E-01	-1.5515E 00
4.2000E 00	2.1065E 00	-7.2682E-01	-1.6249E 00
4.3000E 00	2.1107E 00	-7.1055E-01	-1.6968E 00
4.4000E 00	2.1120E 00	-6.9125E-01	-1.7669E 00
4.5000E 00	2.1107E 00	-6.6906E-01	-1.8349E 00
4.6000E 00	2.1068E 00	-6.4410E-01	-1.9006E 00
4.7000E 00	2.1006E 00	-6.1656E-01	-1.9637E 00
4.8000E 00	2.0923E 00	-5.8661E-01	-2.0239E 00
4.9000E 00	2.0820E 00	-5.5445E-01	-2.0809E 00
5.0000E 00	2.0698E 00	-5.2027E-01	-2.1347E 00
5.1000E 00	2.0560E 00	-4.8430E-01	-2.1849E 00
5.2000E 00	2.0408E 00	-4.4676E-01	-2.2315E 00
5.3000E 00	2.0242E 00	-4.0788E-01	-2.2742E 00
5.4000E 00	2.0066E 00	-3.6789E-01	-2.3130E 00

Table 4.1. Computer output data for X, S_B, and S_K.

TIME	X	SENSB	SENSK
5.5000E 00	1.9880E 00	-3.2703E-01	-2.3478E 00
5.6000E 00	1.9686E 00	-2.8555E-01	-2.3784E 00
5.7000E 00	1.9485E 00	-2.4366E-01	-2.4049E 00
5.8000E 00	1.9280E 00	-2.0162E-01	-2.4271E 00
5.9000E 00	1.9072E 00	-1.5965E-01	-2.4452E 00
6.0000E 00	1.8862E 00	-1.1798E-01	-2.4591E 00
6.1000E 00	1.8651E 00	-7.6815E-02	-2.4688E 00
6.2000E 00	1.8441E 00	-3.6382E-02	-2.4745E 00
6.3000E 00	1.8233E 00	3.1268E-03	-2.4761E 00
6.4000E 00	1.8028E 00	4.1520E-02	-2.4739E 00
6.5000E 00	1.7827E 00	7.8616E-02	-2.4679E 00
6.6000E 00	1.7631E 00	1.1425E-01	-2.4582E 00
6.7000E 00	1.7441E 00	1.4827E-01	-2.4451E 00
6.8000E 00	1.7257E 00	1.8054E-01	-2.4286E 00
6.9000E 00	1.7080E 00	2.1092E-01	-2.4090E 00
7.0000E 00	1.6912E 00	2.3931E-01	-2.3865E 00
7.1000E 00	1.6752E 00	2.6561E-01	-2.3612E 00
7.2000E 00	1.6600E 00	2.8974E-01	-2.3334E 00
7.3000E 00	1.6458E 00	3.1163E-01	-2.3033E 00
7.4000E 00	1.6326E 00	3.3123E-01	-2.2712E 00
7.5000E 00	1.6203E 00	3.4851E-01	-2.2372E 00
7.6000E 00	1.6089E 00	3.6344E-01	-2.2016E 00
7.7000E 00	1.5986E 00	3.7601E-01	-2.1646E 00
7.8000E 00	1.5893E 00	3.8623E-01	-2.1264E 00
7.9000E 00	1.5810E 00	3.9412E-01	-2.0874E 00
8.0000E 00	1.5736E 00	3.9972E-01	-2.0477E 00
8.1000E 00	1.5673E 00	4.0306E-01	-2.0075E 00
8.2000E 00	1.5618E 00	4.0420E-01	-1.9671E 00
8.3000E 00	1.5574E 00	4.0322E-01	-1.9268E 00
8.4000E 00	1.5538E 00	4.0017E-01	-1.8866E 00
8.5000E 00	1.5510E 00	3.9516E-01	-1.8468E 00
8.6000E 00	1.5491E 00	3.8828E-01	-1.8076E 00
8.7000E 00	1.5480E 00	3.7962E-01	-1.7692E 00
8.8000E 00	1.5477E 00	3.6930E-01	-1.7317E 00
8.9000E 00	1.5480E 00	3.5743E-01	-1.6954E 00
9.0000E 00	1.5491E 00	3.4413E-01	-1.6603E 00
9.1000E 00	1.5507E 00	3.2952E-01	-1.6266E 00
9.2000E 00	1.5529E 00	3.1373E-01	-1.5944E 00
9.3000E 00	1.5557E 00	2.9689E-01	-1.5639E 00
9.4000E 00	1.5590E 00	2.7913E-01	-1.5351E 00
9.5000E 00	1.5627E 00	2.6057E-01	-1.5081E 00
9.6000E 00	1.5667E 00	2.4135E-01	-1.4830E 00
9.7000E 00	1.5712E 00	2.2160E-01	-1.4598E 00
9.8000E 00	1.5759E 00	2.0144E-01	-1.4387E 00
9.9000E 00	1.5809E 00	1.8100E-01	-1.4195E 00
1.0000E 01	1.5860E 00	1.6040E-01	-1.4025E 00
1.0100E 01	1.5914E 00	1.3976E-01	-1.3875E 00
1.0200E 01	1.5969E 00	1.1920E-01	-1.3745E 00
1.0300E 01	1.6024E 00	9.8811E-02	-1.3636E 00
1.0400E 01	1.6081E 00	7.8713E-02	-1.3547E 00
1.0500E 01	1.6137E 00	5.8999E-02	-1.3479E 00
1.0600E 01	1.6193E 00	3.9763E-02	-1.3429E 00
1.0700E 01	1.6249E 00	2.1090E-02	-1.3399E 00
1.0800E 01	1.6303E 00	3.0612E-03	-1.3387E 00
1.0900E 01	1.6357E 00	-1.4252E-02	-1.3392E 00

Table 4.1. (Continued)

V. NONLINEAR SYSTEM SENSITIVITY

A. INTRODUCTION

While it is true that no physical systems are exactly linear, often adequate linear approximations can be used. In these cases, the Kokotovic [6] method of sensitivity analysis as developed in the preceding sections of this thesis can be applied. There are many situations, however, for which linear representations are invalid and nonlinear characteristics must be considered.

Figure 5.1 is a block diagram of a nonlinear system containing a linear element represented by a transfer function $K \frac{P(D)}{Q(D)}$, where $D \equiv d/dt$ is the differential operator, and a nonlinear element, N, whose output is denoted by f(e). D is used instead of s in the linear transfer function because the Laplace transform and its inverse are generally not suitable for nonlinear analysis.

The sensitivity to gain-constant variations is developed for the general nonlinear system and then applied to a specific system containing a saturation element as the non-linear component.

B. SENSITIVITY COEFFICIENT FOR A GAIN FACTOR

In order to designate the dependence of the variables in Fig. 5.1 the following notation is used:

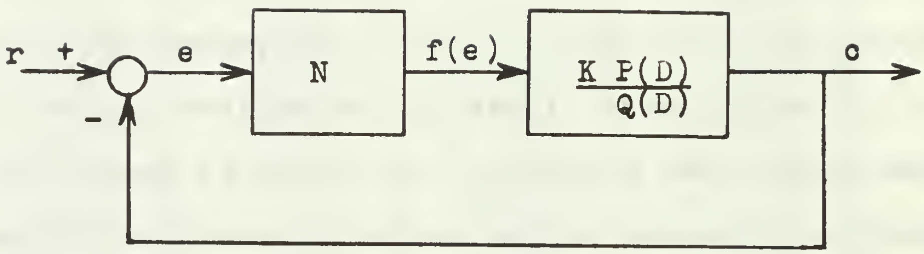


Fig.5.1. Block diagram of nonlinear system.

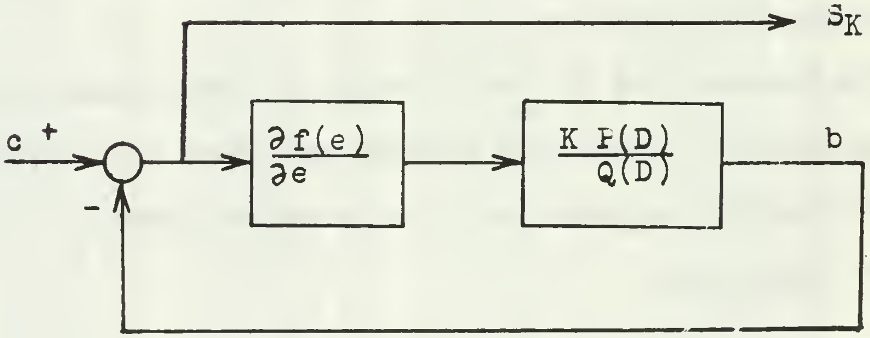


Fig.5.2. Sensitivity model of nonlinear system.

$$r \quad = \quad r(t)$$

$$e \quad = \quad e(t,K) = r(t) - c(t,K)$$

$$f(e) \quad = \quad \text{output of nonlinear element } N$$

$$c \quad = \quad c(t,K) = r(t) - e(t,K)$$

$$x \quad = \quad x(t,K)$$

Let

$$P(D) = a_n \frac{d^n}{dt^n} + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} + \cdots + a_1 \frac{d}{dt} + a_0 \tag{5.1a}$$

$$Q(D) = b_n \frac{d^n}{dt} + b_{n-1} \frac{d^{n-1}}{dt^{n-1}} + \cdots + b_1 \frac{d}{dt} + b_0 \tag{5.1b}$$

From the definition of gain-constant sensitivity coefficient (2.1)

$$S_K = \frac{\partial c}{\partial K/K} = -K \frac{\partial e}{\partial K} \tag{5.2}$$

Using the differential equation for the linear element of the system gives

$$KP(D)\ f\{e(t,K)\} = Q(D)\ c(t,K) = Q(D)\ [r(t)-e(t,K)] \tag{5.4}$$

Taking the logarithmic derivative, $\partial/\partial\ \ell n\ K = K\ \partial/\partial\ K$, of (5.4) and substituting (5.2) yields

$$KP(D)\ f\{e(t,K)\} + K^2P(D)\frac{\partial f\{e(t,K)\}}{\partial K} = -KQ(D)\frac{\partial e(t,K)}{\partial K} \tag{5.5}$$

Rearranging and simplifying (5.5) gives

$$S_K = -K\frac{\partial e}{\partial K} = \frac{KP(D)}{Q(D)}\ f(e) + \frac{K^2P(D)}{Q(D)}\ \frac{\partial f(e)}{\partial K} \tag{5.6}$$

Figure 5.2 is a block diagram of the proposed sensitivity model for the nonlinear system. It will be shown that the describing equation of this model is identical to (5.6).

From Fig. 5.2

$$S_K = \frac{\partial c}{\partial K/K} = c-b \tag{5.7}$$

Substituting (5.2) in (5.7) yields

$$b = c + K\frac{\partial e}{\partial K} \tag{5.8}$$

Applying the transfer function for the linear element of the model gives

$$-K^2P(D)\frac{\partial f(e)}{\partial e}\frac{\partial e}{\partial K} = Q(D)\ b = Q(D)c + KQ(D)\frac{\partial e}{\partial K} \tag{5.9}$$

Substituting (5.4) in (5.9) for Q(D)c and solving for S_K as defined by (5.2) gives

$$S_K = -\frac{K\partial e}{\partial K} = K\frac{P(D)}{Q(D)}f(e) + K^2\frac{P(D)}{Q(D)}\frac{\partial f(e)}{\partial K} \tag{5.10}$$

Equations (5.10) and (5.6) are the same and, hence, the model generates the gain-constant sensitivity coefficient of interest. The sensitivity model block represented by $\frac{\partial f(e)}{\partial e}$ is the instantaneous slope of the nonlinear function f(e). In many nonlinear systems a piecewise-linear approximation can be utilized. The sensitivity model will be the same as that of a linear system except that the gain changes as a function of the error signal in the original nonlinear system. An example for a piecewise-linear saturable element will make this more meaningful.

C. APPLICATION

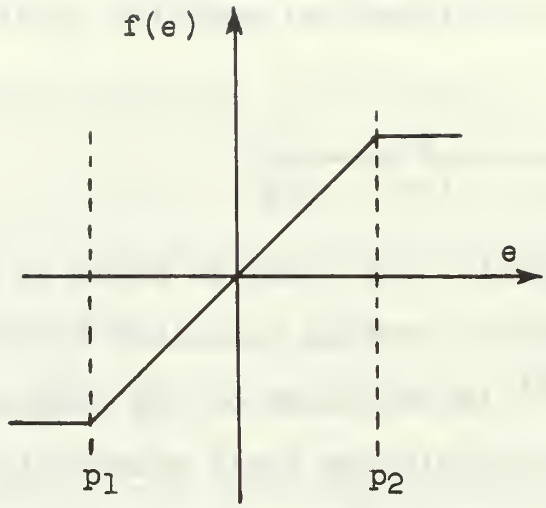
The nonlinear component used as an example will be a limiter with a saturation characteristic as shown in Fig. 5.3. The linear plant used in Section II is used for comparison of the time responses and gain sensitivity coefficients. The transfer function in differential operator notation becomes

$$\frac{KP(D)}{Q(D)} = \frac{50}{D^3 + 10D^2 + 24D}$$

The gain-constant sensitivity model as shown in Fig. 5.2 requires a piecewise-linear function described by Fig. 5.4. It can be seen that $\frac{\partial f(e)}{\partial e}$ is the slope of the nonlinear function f(e) and may be handled by logic statements in the DSL computer program as given in Appendix D.

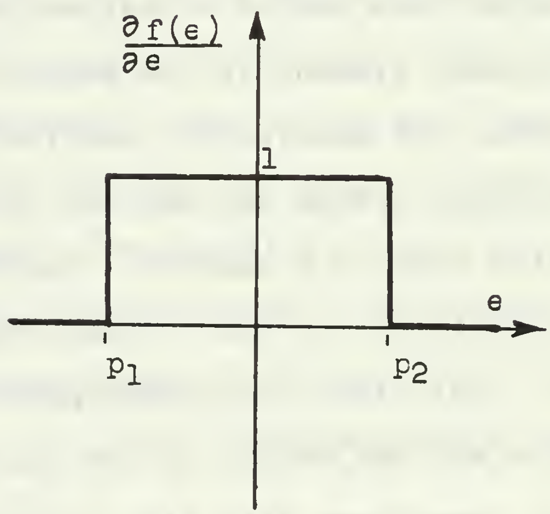
Figure 5.5 shows the time response and the sensitivity function for a unit-step input with a limited amplitude of error signal set at 0.1. The nonlinear element is saturated until 4.8 seconds. During this period of saturation the gain representing the nonlinear element in the sensitivity model is zero, and therefore, the sensitivity coefficient and the output of the nonlinear system are one and the same as shown in Fig. 5.5 for the first 4.8 seconds. Figure 5.6 shows the response and sensitivity for the limited amplitude of $p_1 = -1.0$ and $p_2 = 1.0$. This lets the system perform in a completely linear fashion and the effect of the nonlinear element may be realized by comparison with Fig. 5.5. The nonlinear response is slowed down, the maximum sensitivity to gain is greater but delayed by 3.8 seconds; also, the peak overshoot is greatly reduced.

To verify the sensitivity coefficients for gain variations, check points were chosen in the saturation region,



$$f(e)=p_1, \quad e<p_1$$
$$f(e)=p_2, \quad e>p_2$$
$$f(e)=e, \quad p_1 \le e \le p_2$$

Fig.5.3. Characteristics of nonlinear N.



$$\frac{\partial f(e)}{\partial e} = 1, \quad p_1 \le e \le p_2$$
$$\frac{\partial f(e)}{\partial e} = 0, \quad e<p_1$$
$$\frac{\partial f(e)}{\partial e} = 0, \quad e>p_2$$

Fig.5.4. Characteristics of sensitivity model nonlinearity.

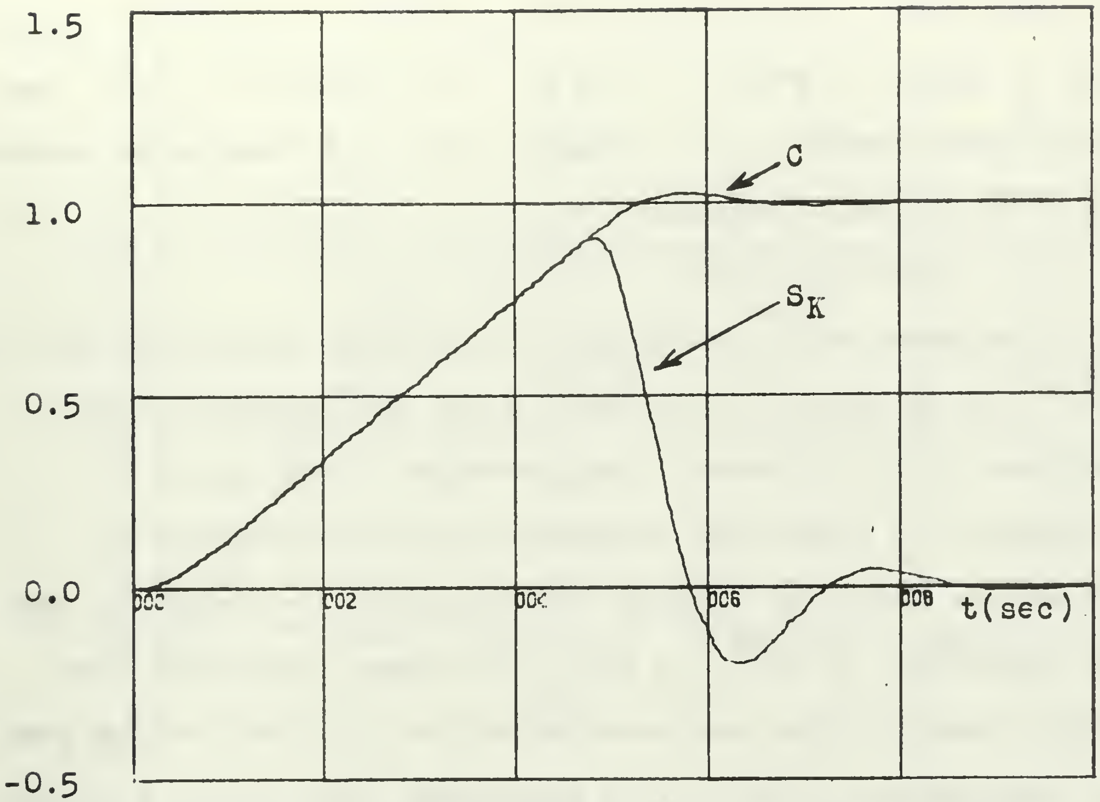


Fig.5.5. Response and sensitivity for nonlinear system with p_1=-0.1 and p_2= 0.1.

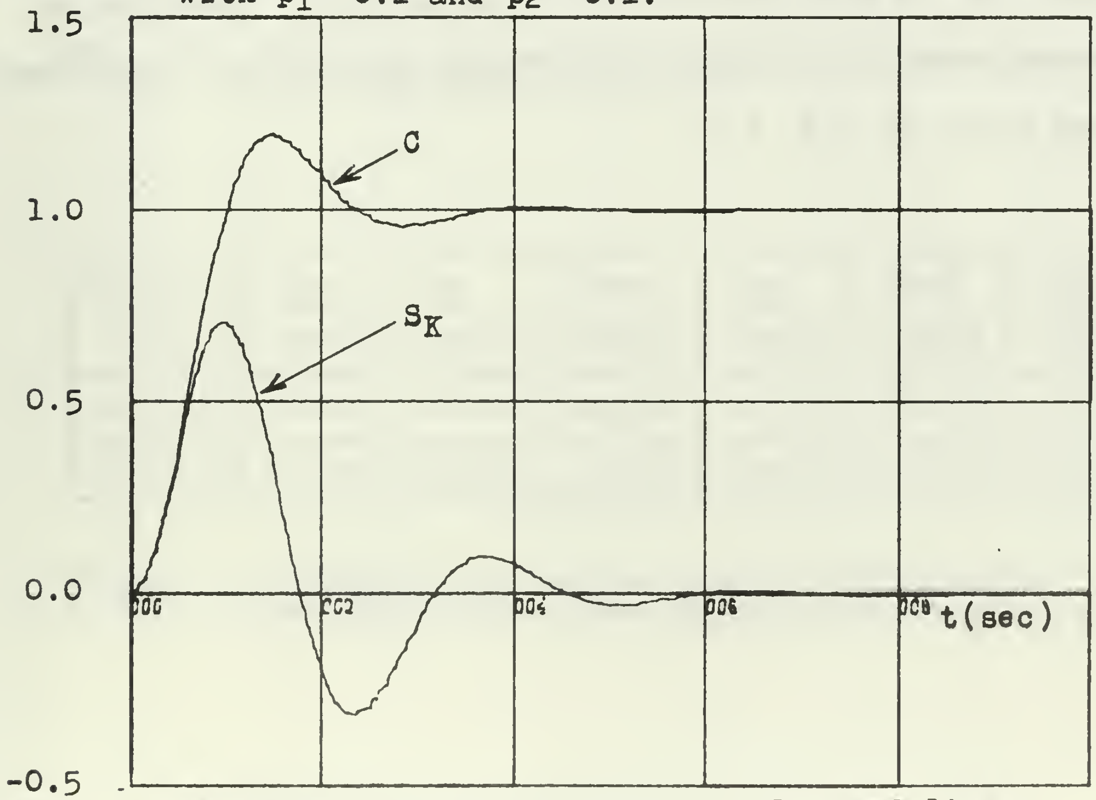


Fig.5.6. Response and sensitivity for unlimited linear system.

peak-positive, and peak-negative portions of the sensitivity curve as shown in Fig. 5.7 for $p_1 = -0.2$ and $p_2 = 0.2$. The approximate change in the response due to a fractional change of 0.1 in the gain parameter,

$$\Delta C \simeq S_K \; \Delta K/K$$

was calculated and compared to the computer output for K = 55 and K = 45 as given in the Table 5.2. The maximum error found was 0.5% for point 2 as presented in Table 5.1.

Figure 5.8 shows the response for 14% variations in gain as was required for the linear system to constrain the peak overshoot to within $\pm$ 0.05. The peak overshoot for the nonlinear system was constrained to $\pm$ 0.014 for the same gain variations. This can be predicted from Fig. 5.8 since at the time of peak overshoot, the sensitivity function is much nearer zero than the sensitivity function for the linear system shown in Fig. 5.7.

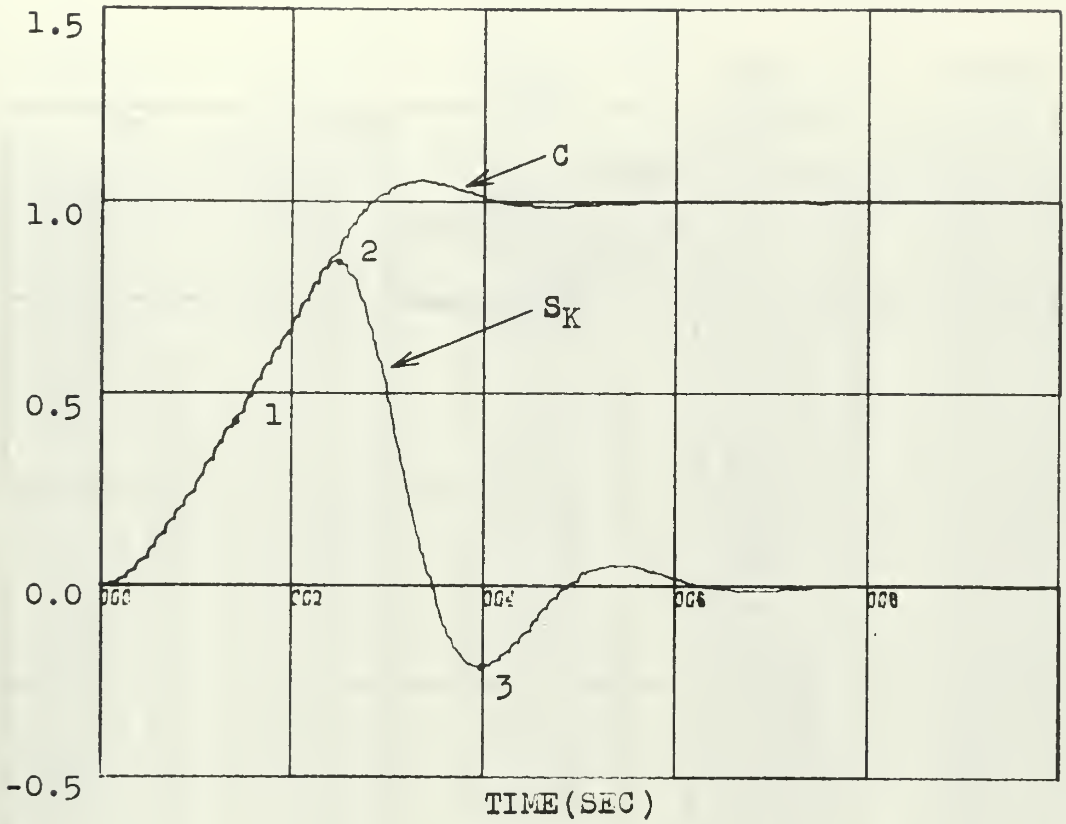


Fig. 5.7. Response and sensitivity for nonlinear system with p_1=-0.2 and p_2= 0.2.

Pt.	T	C_{50}	S_K	$C+\Delta C$	C_{55}	$C-\Delta C$	C_{45}
1	1.5	.45215	.45215	.49737	.49736	.40694	.40693
2	2.5	.86763	.84752	.95238	.94755	.78288	.78127
3	4.0	1.0177	-.21111	.99659	.99673	1.0388	1.0351

Table 5.1. Comparison of actual outputs and outputs predicted from sensitivity coefficients.

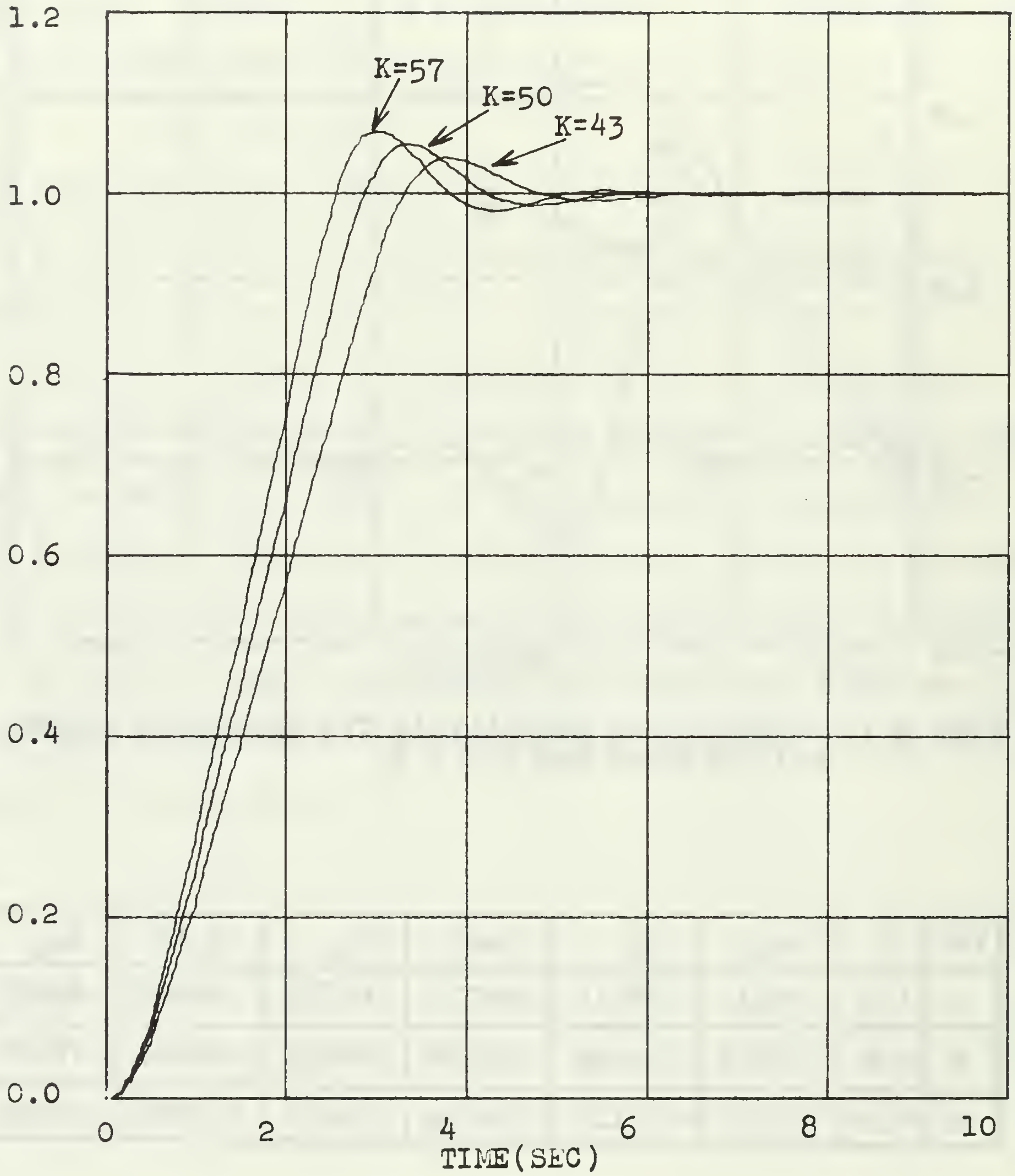


Fig.5.8. Gain tolerance of 14% for overshoot of ±0.014.

		K=50	K=55	K=45
TIME	SENSK	OUT	OUT	OUT
0.0	0.0	0.0	0.0	0.0
1.0000E-01	1.3041E-03	1.3041E-03	1.4345E-03	1.1737E-03
2.0000E-01	8.3024E-03	8.3024E-03	9.1326E-03	7.4721E-03
3.0000E-01	2.2552E-02	2.2552E-02	2.4807E-02	2.0297E-02
4.0000E-01	4.3547E-02	4.3547E-02	4.7902E-02	3.9193E-02
5.0000E-01	7.0099E-02	7.0099E-02	7.7109E-02	6.3089E-02
6.0000E-01	1.0094E-01	1.0094E-01	1.1104E-01	9.0848E-02
7.0000E-01	1.3498E-01	1.3498E-01	1.4847E-01	1.2148E-01
8.0000E-01	1.7132E-01	1.7132E-01	1.8845E-01	1.5419E-01
9.0000E-01	2.0930E-01	2.0930E-01	2.3023E-01	1.8837E-01
1.0000E 00	2.4843E-01	2.4843E-01	2.7328E-01	2.2359E-01
1.1000E 00	2.8837E-01	2.8837E-01	3.1721E-01	2.5953E-01
1.2000E 00	3.2886E-01	3.2886E-01	3.6174E-01	2.9597E-01
1.3000E 00	3.6972E-01	3.6972E-01	4.0670E-01	3.3275E-01
1.4000E 00	4.1085E-01	4.1085E-01	4.5193E-01	3.6976E-01
1.5000E 00	4.5215E-01	4.5215E-01	4.9736E-01	4.0693E-01
1.6000E 00	4.9357E-01	4.9357E-01	5.4292E-01	4.4421E-01
1.7000E 00	5.3507E-01	5.3507E-01	5.8857E-01	4.8156E-01
1.8000E 00	5.7662E-01	5.7662E-01	6.3428E-01	5.1896E-01
1.9000E 00	6.1821E-01	6.1821E-01	6.8003E-01	5.5639E-01
2.0000E 00	6.5983E-01	6.5983E-01	7.2581E-01	5.9385E-01
2.1000E 00	7.0146E-01	7.0146E-01	7.7161E-01	6.3132E-01
2.2000E 00	7.4311E-01	7.4311E-01	8.1742E-01	6.6880E-01
2.3000E 00	7.8476E-01	7.8476E-01	8.6294E-01	7.0628E-01
2.4000E 00	8.2494E-01	8.2641E-01	9.0691E-01	7.4377E-01
2.5000E 00	8.4752E-01	8.6763E-01	9.4755E-01	7.8127E-01
2.6000E 00	8.3906E-01	9.0720E-01	9.8330E-01	8.1877E-01
2.7000E 00	7.9771E-01	9.4366E-01	1.0130E 00	8.5600E-01
2.8000E 00	7.2738E-01	9.7577E-01	1.0361E 00	8.9205E-01
2.9000E 00	6.3457E-01	1.0027E 00	1.0523E 00	9.2573E-01
3.0000E 00	5.2669E-01	1.0239E 00	1.0621E 00	9.5599E-01
3.1000E 00	4.1113E-01	1.0395E 00	1.0661E 00	9.8207E-01
3.2000E 00	2.9468E-01	1.0496E 00	1.0652E 00	1.0035E 00
3.3000E 00	1.8319E-01	1.0548E 00	1.0603E 00	1.0203E 00
3.4000E 00	8.1344E-02	1.0558E 00	1.0526E 00	1.0324E 00
3.5000E 00	-7.3481E-03	1.0533E 00	1.0431E 00	1.0403E 00
3.6000E 00	-8.0616E-02	1.0483E 00	1.0328E 00	1.0444E 00
3.7000E 00	-1.3732E-01	1.0416E 00	1.0224E 00	1.0453E 00
3.8000E 00	-1.7736E-01	1.0338E 00	1.0126E 00	1.0436E 00
3.9000E 00	-2.0147E-01	1.0256E 00	1.0040E 00	1.0401E 00
4.0000E 00	-2.1111E-01	1.0177E 00	9.9673E-01	1.0351E 00
4.1000E 00	-2.0823E-01	1.0103E 00	9.9112E-01	1.0294E 00
4.2000E 00	-1.9509E-01	1.0038E 00	9.8717E-01	1.0234E 00
4.3000E 00	-1.7412E-01	9.9845E-01	9.8479E-01	1.0174E 00
4.4000E 00	-1.4774E-01	9.9425E-01	9.8382E-01	1.0118E 00
4.5000E 00	-1.1826E-01	9.9124E-01	9.8406E-01	1.0067E 00
4.6000E 00	-8.7735E-02	9.8934E-01	9.8526E-01	1.0024E 00
4.7000E 00	-5.7953E-02	9.8844E-01	9.8714E-01	9.9881E-01
4.8000E 00	-3.0345E-02	9.8839E-01	9.8947E-01	9.9607E-01
4.9000E 00	-5.9862E-03	9.8901E-01	9.9200E-01	9.9412E-01
5.0000E 00	1.4402E-02	9.9014E-01	9.9454E-01	9.9289E-01
5.1000E 00	3.0435E-02	9.9161E-01	9.9693E-01	9.9229E-01
5.2000E 00	4.2025E-02	9.9327E-01	9.9905E-01	9.9222E-01
5.3000E 00	4.9333E-02	9.9497E-01	1.0008E 00	9.9257E-01
5.4000E 00	5.2723E-02	9.9662E-01	1.0022E 00	9.9323E-01

Table 5.2. Computer output data for nonlinear system outputs and sensitivity.

		K=50	K=55	K=45
TIME	SENSK	OLT	OLT	OLT
5.5000E 00	5.2703E-02	9.9813E-01	1.0031E 00	9.9411E-01
5.6000E 00	4.9877E-02	9.9944E-01	1.0037E 00	9.9511E-01
5.7000E 00	4.4889E-02	1.0005E 00	1.0040E 00	9.9616E-01
5.8000E 00	3.8392E-02	1.0013E 00	1.0039E 00	9.9718E-01
5.9000E 00	3.1001E-02	1.0019E 00	1.0036E 00	9.9813E-01
6.0000E 00	2.3272E-02	1.0023E 00	1.0031E 00	9.9899E-01
6.1000E 00	1.5682E-02	1.0024E 00	1.0026E 00	9.9971E-01
6.2000E 00	8.6141E-03	1.0024E 00	1.0019E 00	1.0003E 00
6.3000E 00	2.3564E-03	1.0023E 00	1.0013E 00	1.0007E 00
6.4000E 00	-2.8982E-03	1.0020E 00	1.0007E 00	1.0011E 00
6.5000E 00	-7.0448E-03	1.0017E 00	1.0002E 00	1.0012E 00
6.6000E 00	-1.0060E-02	1.0013E 00	9.9980E-01	1.0013E 00
6.7000E 00	-1.1984E-02	1.0010E 00	9.9946E-01	1.0013E 00
6.8000E 00	-1.2908E-02	1.0006E 00	9.9923E-01	1.0013E 00
6.9000E 00	-1.2962E-02	1.0003E 00	9.9909E-01	1.0011E 00
7.0000E 00	-1.2303E-02	1.0001E 00	9.9903E-01	1.0010E 00
7.1000E 00	-1.1094E-02	9.9985E-01	9.9904E-01	1.0008E 00
7.2000E 00	-9.5025E-03	9.9969E-01	9.9912E-01	1.0006E 00
7.3000E 00	-7.6836E-03	9.9958E-01	9.9923E-01	1.0005E 00
7.4000E 00	-5.7786E-03	9.9951E-01	9.9937E-01	1.0003E 00
7.5000E 00	-3.9067E-03	9.9949E-01	9.9952E-01	1.0001E 00
7.6000E 00	-2.1646E-03	9.9950E-01	9.9967E-01	1.0000E 00
7.7000E 00	-6.2460E-04	9.9953E-01	9.9982E-01	9.9993E-01
7.8000E 00	6.6608E-04	9.9959E-01	9.9994E-01	9.9986E-01
7.9000E 00	1.6829E-03	9.9966E-01	1.0000E 00	9.9981E-01
8.0000E 00	2.4202E-03	9.9973E-01	1.0001E 00	9.9978E-01
8.1000E 00	2.8889E-03	9.9981E-01	1.0002E 00	9.9977E-01
8.2000E 00	3.1124E-03	9.9988E-01	1.0002E 00	9.9977E-01
8.3000E 00	3.1238E-03	9.9994E-01	1.0002E 00	9.9978E-01
8.4000E 00	2.9615E-03	9.9999E-01	1.0002E 00	9.9980E-01
8.5000E 00	2.6658E-03	1.0000E 00	1.0002E 00	9.9983E-01
8.6000E 00	2.2785E-03	1.0001E 00	1.0002E 00	9.9986E-01
8.7000E 00	1.8387E-03	1.0001E 00	1.0002E 00	9.9989E-01
8.8000E 00	1.3797E-03	1.0001E 00	1.0001E 00	9.9992E-01
8.9000E 00	9.3007E-04	1.0001E 00	1.0001E 00	9.9995E-01
9.0000E 00	5.1326E-04	1.0001E 00	1.0000E 00	9.9997E-01
9.1000E 00	1.4567E-04	1.0001E 00	1.0000E 00	9.9999E-01
9.2000E 00	-1.6022E-04	1.0001E 00	9.9999E-01	1.0000E 00
9.3000E 00	-4.0054E-04	1.0001E 00	9.9997E-01	1.0000E 00
9.4000E 00	-5.7411E-04	1.0001E 00	9.9995E-01	1.0000E 00
9.5000E 00	-6.8283E-04	1.0000E 00	9.9994E-01	1.0000E 00
9.6000E 00	-7.3338E-04	1.0000E 00	9.9994E-01	1.0000E 00
9.7000E 00	-7.3433E-04	1.0000E 00	9.9994E-01	1.0000E 00
9.8000E 00	-6.9433E-04	1.0000E 00	9.9994E-01	1.0000E 00
9.9000E 00	-6.2269E-04	9.9999E-01	9.9995E-01	1.0000E 00
1.0000E 01	-5.2989E-04	9.9998E-01	9.9996E-01	1.0000E 00
1.0100E 01	-4.2456E-04	9.9998E-01	9.9997E-01	1.0000E 00
1.0200E 01	-3.1644E-04	9.9998E-01	9.9998E-01	1.0000E 00
1.0300E 01	-2.1058E-04	9.9998E-01	9.9999E-01	1.0000E 00
1.0400E 01	-1.1283E-04	9.9998E-01	9.9999E-01	1.0000E 00
1.0500E 01	-2.7299E-05	9.9998E-01	1.0000E 00	1.0000E 00
1.0600E 01	4.3333E-05	9.9998E-01	1.0000E 00	1.0000E 00
1.0700E 01	9.8586E-05	9.9998E-01	1.0000E 00	9.9999E-01
1.0800E 01	1.3816E-04	9.9999E-01	1.0000E 00	9.9999E-01
1.0900E 01	1.6284E-04	9.9999E-01	1.0000E 00	9.9999E-01

Table 5.2. (Continued)

VI. CONCLUSIONS

The technique of generating sensitivity functions for a basic linear feedback control system using the concept of a sensitivity model is developed. This model is a replica of the original system and is cascaded with it. All sensitivity functions are generated simultaneously with the time response by use of the IBM/360 Digital Simulation Program.

Sensitivity of a differential equation with respect to its coefficients is investigated. Computer graphs show the differences in the influence of the coefficients on the solution of the differential equation. The coefficients of the lower-order terms have the greatest influence but the effect is obtained with the greatest time delay.

A theoretical sensitivity model is developed for a nonlinear control system. Instead of a replica of the original control system as required for the linear model, the nonlinear element is replaced by a component with the instantaneous gain equal the slope of the nonlinear function. An example illustrates the procedure for a nonlinearity due to a saturation element.

Sensitivity analysis indicates that proper interpretation of the sensitivity function results in accurate estimates for changes in the system response due to parameter variations. Tolerance limits of parameter variations are determined and performance improved by application of sensitivity coefficients.

APPENDIX A

DSL/360 DIGITAL SIMULATION PROGRAM FOR BASIC SYSTEM

```
TITLE BASIC SERVO SENSITIVITY COEFFICIENTS

INTEG RKS
STORAG IC(3),NUM(1),DEN(4)
TABLE IC(1-3)=4*0,NUM(1)=1.0,DEN(1-4)=1.,10.,24.,0.
CONTRL FINTIM=10.,DELT=.1
PARAM IN=1.,F=1.,K=50.

* SOLUTION OF BASIC SYSTEM FOR TIME RESPONSE

      ERROR1=IN-F*OUT
      OUT=TRNFR(0,3,IC,NUM,DEN,K*ERROR1)

* SOLUTION OF MODEL FOR SENSITIVITY COEFFICIENTS
* SENSK=SENSITIVITY TO FORWARD GAIN K
* SENSF=SENSITIVITY TO FEEDBACK GAIN F

      SENSK=OUT+SENSF
      X=TRNFR(0,3,IC,NUM,DEN,K*SENSK)
      SENSF=X*(-F)

PRINT .1,OUT,SENSK,SENSF
PREPAR .1,OUT,SENSK,SENSF
GRAPH SAME,TIME,OUT,SENSK,SENSF
PRPLOT ONLY
END
STOP
```

APPENDIX B

DSL/360 DIGITAL SIMULATION PROGRAM FOR MULTILOOP SYSTEM

```
TITLE MULTILOOP SERVO SENSITIVITY COEFFICIENTS

INTEG RKS
CONTRL FINTIM=10.0,DELT=.01
PARAM K1=40.,K2=1.,K3=1.,K4=2.,FGAIN=1.,IN=1.,TAU1=4.,...
      TAU4=1.

* SOLUTION OF MULTILOOP SYSTEM FOR TIME RESPONSE

      ERROR1=IN-FGAIN*OUT
      D=EXCIT*(-K4)*TAU4
      FED1=D
      FED2=REALPL(0.,TAU4,D)
      FEDB=FED1-FED2
      ERROR2=ERROR1+FEDB
      EXCIT=REALPL(0.,TAU1,K1*ERROR2)
      GEN=REALPL(0.,2.,K2*EXCIT)
      OUT=REALPL(0.,1.,K3*GEN)

* SOLUTION OF MODEL FOR SENSITIVITY COEFFICIENTS

      SENSK3=OUT-FGAIN*Z
      DX=X*(-K4)*TAU4
      FEDX1=DX
      FEDX2=REALPL(0.,TAU4,DX)
      SENSK4=FEDX1-FEDX2
      SENST4=REALPL(0.,TAU4,SENSK4)
      SENSK1=SENSK3+SENSK4
      P=-SENSK1
      Q=REALPL(0.,TAU1,P)
      SENST1=P-Q
      X=REALPL(0.,4.,K1*SENSK1)
      Y=REALPL(0.,2.,K2*X)
      Z=REALPL(0.,1.,K3*Y)

PRINT  .1,OUT,SENSK3,SENSK1,SENST1,SENSK4,SENST4
PREPAR .1,OUT,SENSK3,SENSK1,SENST1,SENSK4,SENST4
GRAPH  SAME,TIME,OUT,SENSK3,SENSK1,SENST1,SENSK4,SENST4
GRAPH  SAME,TIME,SENSK1,SENST1
PRPLOT ONLY
END
STOP
```

APPENDIX C

DSL/360 DIGITAL SIMULATION PROGRAM FOR SIXTH ORDER DIFFERENTIAL EQUATION

```
TITLE SIXTH ORDER DIFFERENTIAL EQUATION SENSITIVITY MODEL

INTEG RKS
CONTRL FINTIM=50.,DELT=.05
PARAM A=1.,B=3.4,C=11.4,D=9.4,E=7.4,F=.575,G=.452

* SOLUTION OF DE FOR TIME RESPONSE

      IN=STEP(0)
      XD6=(-B*XD5)+(-C*XD4)+(-D*XD3)+(-E*XD2)+(-F*XD1)...
      +(-G*X)+IN
      XD5=INTGRL(0.,XD6)
      XD4=INTGRL(0.,XD5)
      XD3=INTGRL(0.,XD4)
      XD2=INTGRL(0.,XD3)
      XD1=INTGRL(0.,XD2)
      X=INTGRL(0.,XD1)

* SOLUTION OF MODEL FOR SENSITIVITY COEFFICIENTS

      SENSG=-G*SX
      SENSF=-F*SXD1
      SENSE=-E*SXD2
      SENSD=-D*SXD3
      SENSC=-C*SXD4
      SENSB=-B*SXD5
      SXD6=SENSB+SENSC+SENSD+SENSE+SENSF+SENSG+X
      SXD5=INTGRL(0.,SXD6)
      SXD4=INTGRL(0.,SXD5)
      SXD3=INTGRL(0.,SXD4)
      SXD2=INTGRL(0.,SXD3)
      SXD1=INTGRL(0.,SXD2)
      SX=INTGRL(0.,SXD1)

PRINT  .1,X,SENSB,SENSC,SENSD,SENSE,SENSF,SENSG
PREPAR .1,X,SENSB,SENSC,SENSD,SENSE,SENSF,SENSG
GRAPH SAME,TIME,X,SENSB,SENSC,SENSD,SENSE,SENSF,SENSG
PRPLOT ONLY
END
STOP
```

APPENDIX D

DSL/360 DIGITAL SIMULATION PROGRAM FOR NONLINEAR STSTEM

```
TITLE NONLINEAR SYSTEM SENSITIVITY

INTEG RKS
STORAG IC(3),B(4)
TABLE IC(1-3)=3*0,B(1-4)=1.,10.,24.,0.
CONTRL FINTIM=12.,DELT=.1
PARAM K=50.,P1=-.2,P2=.2

* SOLUTION OF NONLINEAR SYSTEM FOR TIME RESPONSE

      IN=STEP(0)
      ERROR=IN-OUT
      Y=LIMIT(P1,P2,ERROR)
      OUT=DEQN(3,IC,B,K*Y)

* SOLUTION OF MODEL FOR SENSITIVITY COEFFICIENTS
* SENSK=SENSITIVITY TO FORWARD GAIN K

NOSORT
      IF(ERROR.GT.P2.OR.ERROR.LT.P1)GO TO 1
      M=OUT-N
      GO TO 2
    1 M=0
    2 CONTINUE
      N=DEQN(3,IC,B,K*M)
      SENSK=OUT-N

PRINT .1,ERROR,OUT,SENSK
PREPAR .1,OUT,SENSK
GRAPH SAME,TIME,OUT,SENSK
PRPLOT ONLY
END
STOP
```

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13. ABSTRACT

A technique for the use of a digital computer for simultaneous generation of feedback control system transient response and sensitivity to parameter variations is developed. The sensitivity model for a basic linear control system is a cascaded replica of the original system with the sensitivity pickoff points corresponding to the error and feedback signals of the original system. This is convenient for computer analysis with DSL/360. For systems with nonlinear elements, the sensitivity model is implemented by making the instantaneous gain equal to the slope of the nonlinear function. Sensitivity analysis is utilized for improvement of system performance, determining parameter tolerances, and predicting changes in the response due to parameter variations.

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